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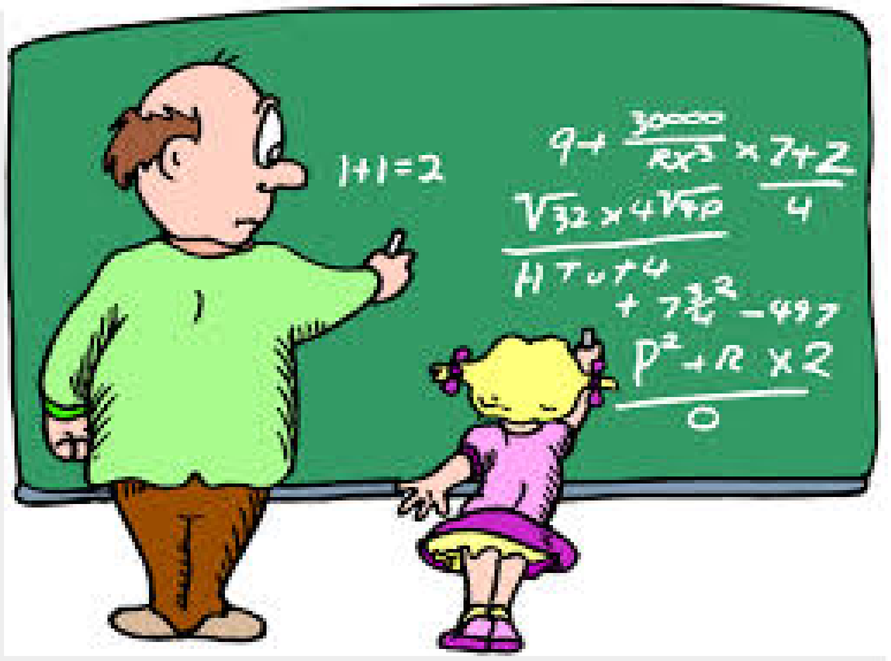
Providing structure in mathematical tasks and keeping challenge: but how?

- Motivation, self-regulation and achievements
- Enriching regular lessons
- Mathematics and History education
- Study the effect of the intervention



Research on potentially cognitive talented students (16/17 years old) (2013-2015)

CONTEXT OF THIS SESSION

Central research questions

- What are the effects of the learning arrangements on students’ motivation, self-regulated learning and achievements in history and mathematics?
- What is the effect of the amount of structure on students’ motivation, self-regulated learning, learning processes, and achievements?
- What is the effect of the constitution of the group during collaborative learning tasks on students’ motivation, self-regulated learning, learning processes and achievements

Research questions - design phase

- What are essential task features that allows us to differentiate between tasks (more structured and less structured)?
- How can we adapt or develop mathematical tasks in order to meet these essential features?

Goal for this session

- To get more insight in characteristics of learning environments that can support meaningful learning of mathematics. In particular with regard to mathematization, autonomy support and structure
- Try to define structure as task feature and varying the amount of structure within the task
- To get known with examples and some guidelines that can support selection/adaptation/design of learning activities
- To identify and discuss (unsolved) problems and issues around achieving the previous goals

Plan

1. What we do know and what we don't know about autonomy aupport and structure → Guidelines
2. Structure as task feature
   - Task 1 → designing for autonomy support and structure
   - Task 2 → structuring and unstructuring a learning task
   - Task 3 → discussing/adapting mathematical tasks
3. Observing → Students perception of the tasks
4. Revising guidelines → trying to answer the initial questions

LEARNING ACTIVITIES FOR

AUTONOMY SUPPORT & STRUCTURE

What means to you autonomy supportive learning environments ?

And, what is structure support?

Previous research

Theoretical perspectives

Examples

LEARNING ACTIVITIES FOR

AUTONOMY SUPPORT & STRUCTURE

Theoretical perspectives

- Self-Determination Theory (SDT) , (Reeves, 2009)

  In a broader sense (how people learn), it is appropriate to define aspects of the learning environment that contribute to the development of students' motivation, self-regulation and cognitive ability ***(autonomy support);*** *also, it provides directions to study the relation between* ***provision of structure***

- Realistic Mathematics Education (RME) , (Freudenthal, Treffers, Gravemeijer)

  With regard to mathematics education, RME provides a framework for what is mathematics and ***mathematics learning****; namely, it guides us in establishing learning goals, the choice of learning activities, analysing mathematical growth*

Autonomy support

(Stroet et al, 2013)

- Associated with the need for autonomy, which finds its' origin in the inherent desire people have to be causal agents and to act in accordance with their sense of self.
- For students to feel autonomous, it is crucial to experience their engagement in learning as a self-chosen act reflecting their own authentic needs and values.
- Feeling autonomous is not the same as feeling independent of others; autonomously initiated actions can be initiated either independently or in response to a request of significant others

Research on autonomy support

(Su & Reeve, 2010)

- Meta-analyse on the effectiveness of training intervention programs
- Identify set of conditions (five elements of autonomy support) that allowed these interventions to be most effective
  - Nurture inner motivational resources
  - Provide meaningful rationales
  - Offer choices
  - Use non-controlling language
  - Acknowledge negative feelings
- One recommendation to practitioners in designing an autonomy supportive intervention is to include multiple elements of autonomy support

Example providing meaningful rationales

(Reeves, 2009)

Your paper is due on Monday.
Today, we are going to the school library.
In the library, you will find information from books and Internet sites to use for your paper.
Don't waste your time; don't goof off; make sure to get your work done. In the library, you may work by yourself or with a partner.

Requests and choices without explanatory rationals

Requests and choices WITH explanatory rationals

Your paper is due on Monday. As a way of helping you write a well-researched paper, we are going to where the information is—the school library. The reason we are going to the library is to find the information you need from books and Internet sites. While there, you may be tempted to goof off, but students in the past have found that a trip to the library was a crucial part of writing an excellent paper. To help you write your best possible paper, you may work in the way you wish—by yourself or with a partner.

Example digital learning task with autonomy support

(Van Loon et al, 2012)

Several elements of autonomy support:

Interesting and challenging, choices, explanatory rational, non-directive language

- Introduction
- Instruction
- Roadmap
- Sources
  - Why is there advertising?
  - Advertising!
  - How to make advertising?
- Advertisments
  - Laundry detergent
  - Candy
  - Supermarket
- Template to present findings
  - Word
  - PowerPoint
- Assessment

Advertis…

Introdu…

On the TV or in magazines, you often encounter advertising. For example, you may observe advertising for sweets, toys, games or cars. By advertising, businesses hope that you buy more. Many companies are involved in the advertising enterprise. But advertising is (very) expensive. It is, therefore, important that an advertising strategy works well and encourages customers to make the purchase upon seeing the advertisement.

Would you be able to design a good advertisement? In this task, you are a creator of advertisements. You are going to decide what good advertising is and identify the tricks used in advertisements to ensure that customers really do buy more.

Structure provision

- Students differ from each other in their learning needs. E.g. students with high ability may have a stronger need for autonomy and less for structure; underachievers have been found to have low self-efficacy and may have a need for more structure.
- By *structure* we refer to Self Determination Theory which defines it as having three main components:
  - clear goals and expectations
  - appropriate strategies
  - guidance to solve the tasks and clear procedures to be followed

Structured learning environment

- A *structured learning environment* contribute to feelings of competence which are needed for engaging in challenging tasks, problem solving and creative thinking
- *Too much structure* can also lead that students acquire a narrow view of scientific inquiry where the thinking is characteristically rote and low level (Hume & Coll, 2008)
- *Research so far:*
  - often focus on autonomy support and few on structure
  - research on structure support regards mostly teacher support
  - often research on learning activities regard digital learning activities, inquiry based learning, collaborative scripts
  - specifically from mathematics: problem solving/posing

EXAMPLES

Example teacher structure support

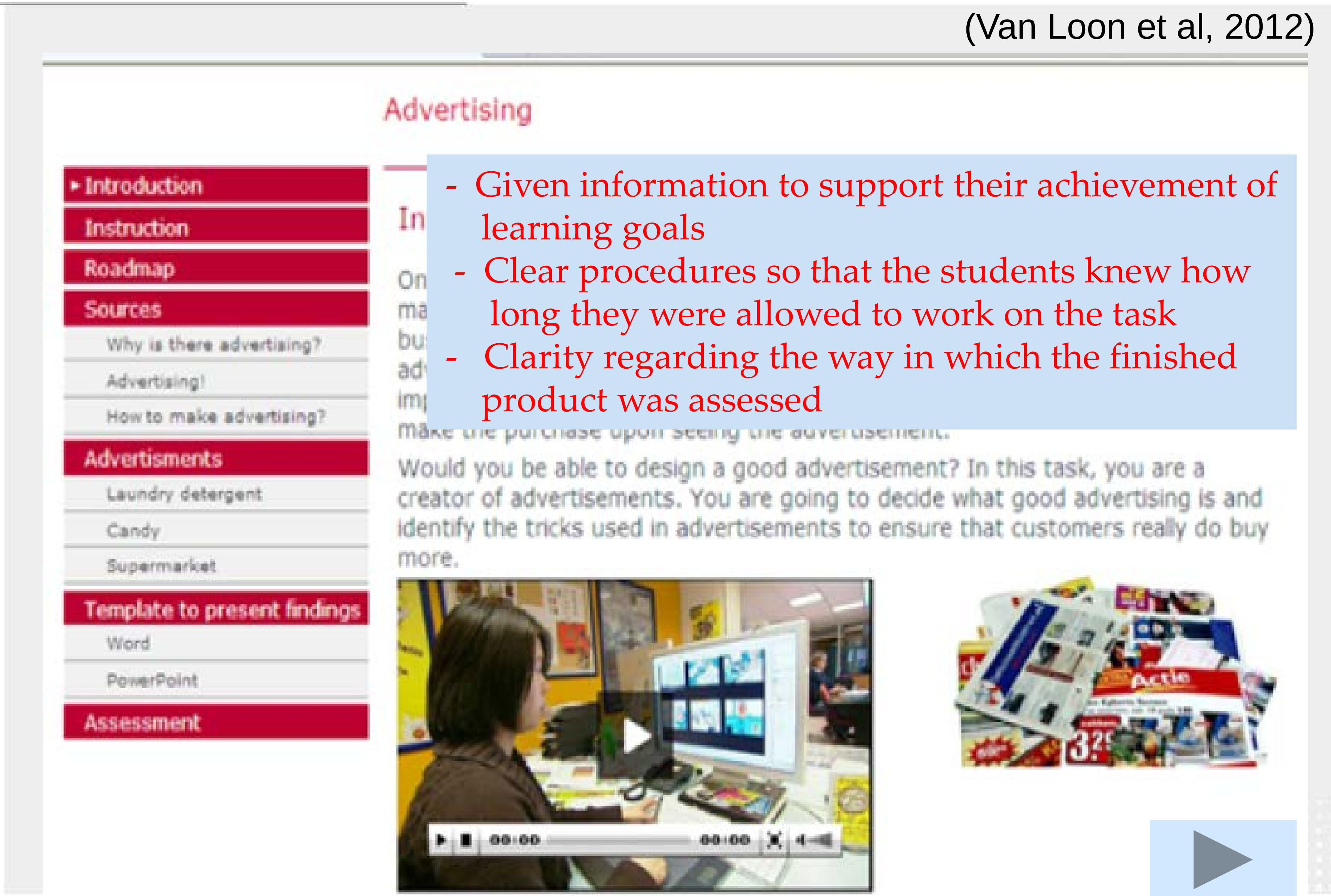
(Several researchers in Jang et al, 2010)

- Teacher-provided structure has been studied extensively within the *classroom management literature*: keeping students on task, managing their behavior, and avoiding chaos during transitions
- Teacher-provided structure *from a motivational point of view* helps students to develop a sense of perceived control over school outcomes—that is, to develop perceived competence
- What structured teachers say and do can be characterized by three categories of instructional behavior:
  - (a) present clear, understandable, explicit, and detailed directions
  - (b) offer a program of action to guide students' ongoing activity
  - (c) offer constructive feedback on how students can gain control

It is only with moderate structure
—some supervision but not totalitarian supervision —
that students learn both prerequisite skills and
the experience of personal causation that promotes engagement

Example digital learning task structured guidance

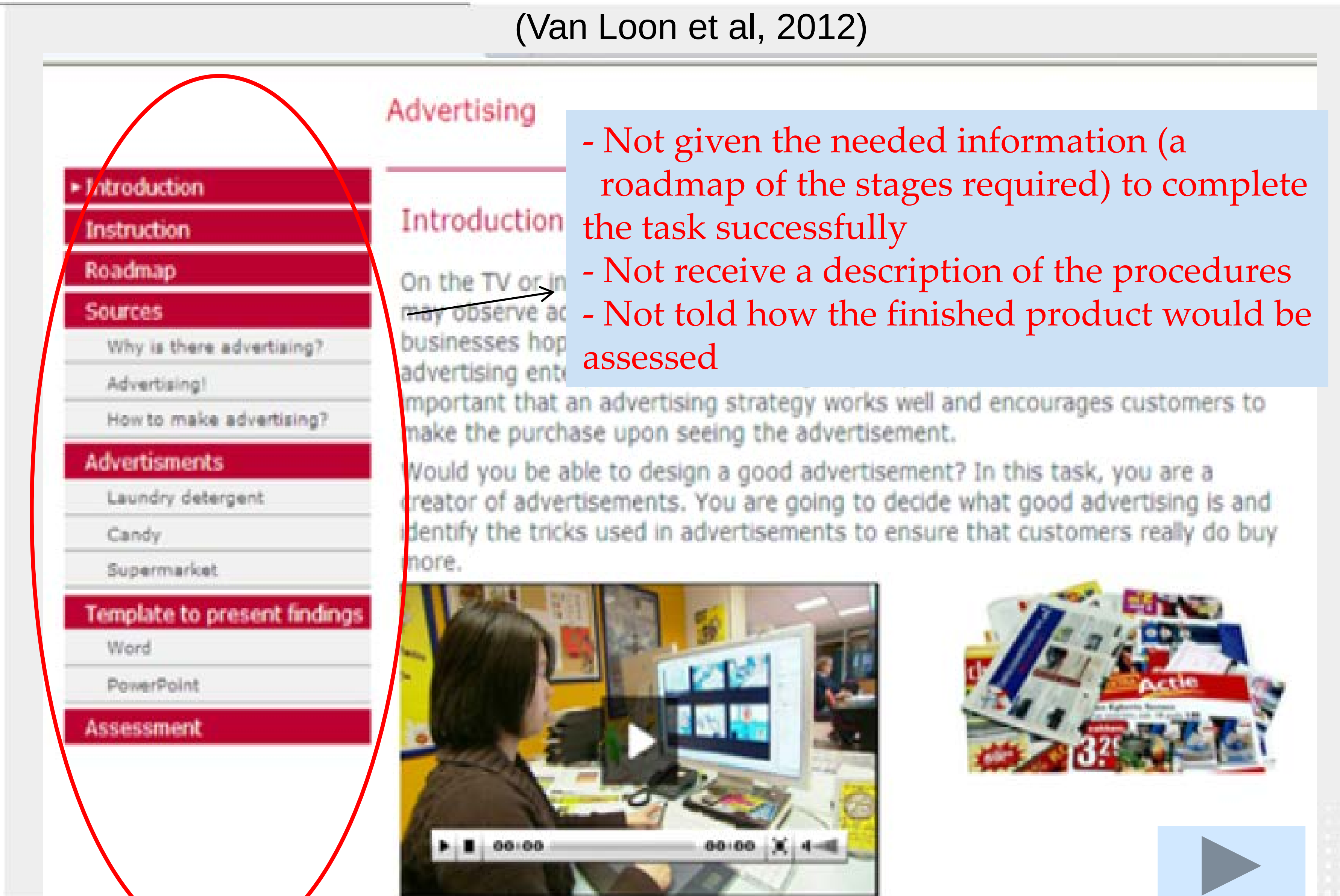
(Van Loon et al, 2012)



- Given information to support their achievement of learning goals
- Clear procedures so that the students knew how long they were allowed to work on the task
- Clarity regarding the way in which the finished product was assessed

Example digital learning task no structured guidance

(Van Loon et al, 2012)



- Not given the needed information (a roadmap of the stages required) to complete the task successfully
- Not receive a description of the procedures
- Not told how the finished product would be assessed

Comparison of the 'amount'of structure

In terms of autonomy support
in both situations students did have control over the content,
the relevance of the task was explained and language was non-directive
In terms of structure

Structured guidance	No structured guidance
given information to support achievement of the learning goal	no information to insure their achievement of the learning goal
clarity regarding the way in which the finished product would be assessed	not told how the finished product would be assessed
clear procedures about what they could do and when they had finished their work	did not receive a description of the procedures and they did not know how long they were allowed to work on the task

Varying structure

Variation on:

- The amount/kind/quality of information given to achieve the learning goal
- Clarity/non clarity on criteria of assessment
- The specificity/non specificity of allowed procedures and spend time

Structured guidance	No structured guidance
give information to support achievement of the learning goal	no information to insure their achievement of the learning goal
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Structure als task feature

Variation on:

- The amount/kind/quality information given to achieve the learning goal
- Clarity/not clarity on criteria of assessment
- The specificity/not specificity of allowed procedures and spend time

Structured	**No structured**
given information to support achievement of the learning goal	no information to insure their achievement of the learning goal
clarity regarding the way in which the finished product would be assessed	not told how the finished product would be assessed
clear procedures about what they could do and when they had finished their work	did not receive a description of the procedures and they did not know how long they were allowed on the task

Structure in problem solving

- Distinction between well-structured problems and ill structured problems (Frederiksen, 1984; Jonassen, 1997).
  - *well-structured problems:* all elements of the problem are presented,
  - *ill structured problems :* unknown elements, multiple solutions (or no solution at all), multiple criteria for evaluating solutions
- Provision of structure regards:
  - Use of worked out examples: description of the entire solution procedure.
  - Use of process steps : description of a general, systematic approach to problem solving that students should follow, but students still have to work-out each step themselves (Nievelstein van Gog, van Dijck & Boshuizen 2013).
- This use of 'structure' seems more related to teaching support than as task feature

Structure in problem posing

- Litrerature on problem posing

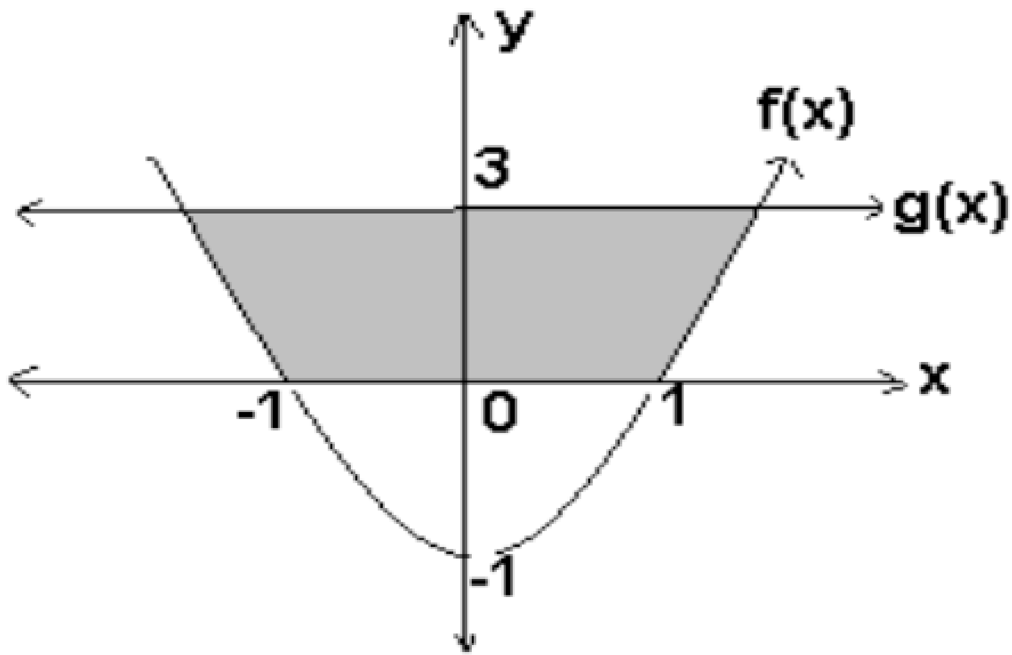
  Stoyanova & Ellerton, 1996; Stoyanova,1998).
  - *semi-structured problem posing situations*
  - *structured problem posing situations*
  - *free problem posing situations*

Example semi-structured problem posing situation

Akay & Boz (2009)

"In this activity, the students were provided with semi-structured mathematical expression. In this expression the actual component is not certain(…)"

Activity 3. As can be seen in the figure below there is a region bounded by the parabolas $f(x)$ and line $g(x)$ and the axes. Pose a problem related to this figure.



Example structured problem posing situation

Akay & Boz (2009)

In this activity without changing the mathematical components, by changing the integers m and n new problems can be posed

Activity 9: $m, n \in Z$ compute $\int \sin^n(x)\cos^m(x)dx = ?$ "Pose different problems depending on the integers m and n. Later try to solve each problem you posed. By looking at your solutions can you get a generalization?"

Guidelines

- Designing an *autonomy supportive intervention* includes multiple elements (nurture inner motivational resources; provide meaningful rationales; offer choices; use non-controlling language; acknowledge negative feelings)

- *Structure* has three main components: clear goals and expectations, appropriate strategies, guidance to solve the tasks and clear procedures to be followed

- *Structure as task feature.* Variation on:
  - The amount/kind/quality of information given to achieve the learning goal
  - Clarity/non clarity on criteria of assessment
  - The specificity/non specificity of allowed procedures and time to spend on task

Varying structure

Structured	Semi-structured	No structured
Given information to support achievement of the learning goal		No information is given to insure their achievement of the learning goal
Clarity regarding the end product and in which way it will be assessed		No clarity regarding the end product nor in which way it will be assessed
Clarity about what they could do and the procedures that they can follow	Clarity about what is expected but no procedure is given to achieve it	No clarity about what they could do nor is given a description of the procedures that they could follow

ACTIVITIES

STRUCTURE AS TASK FEATURE

Task 1

Designing for autonomy support and structure

- Example: the train travel
- Discussion
- Two versions and examples of students answers

Integrals

- Traditionally they are introduced in a formal way using formal notation (16/17 years old students in pre universitary education)
- As result students develop a superficial concept image of integrals and students have difficulty to use and apply integrals in problem situations and in other subjects.

Integraal:

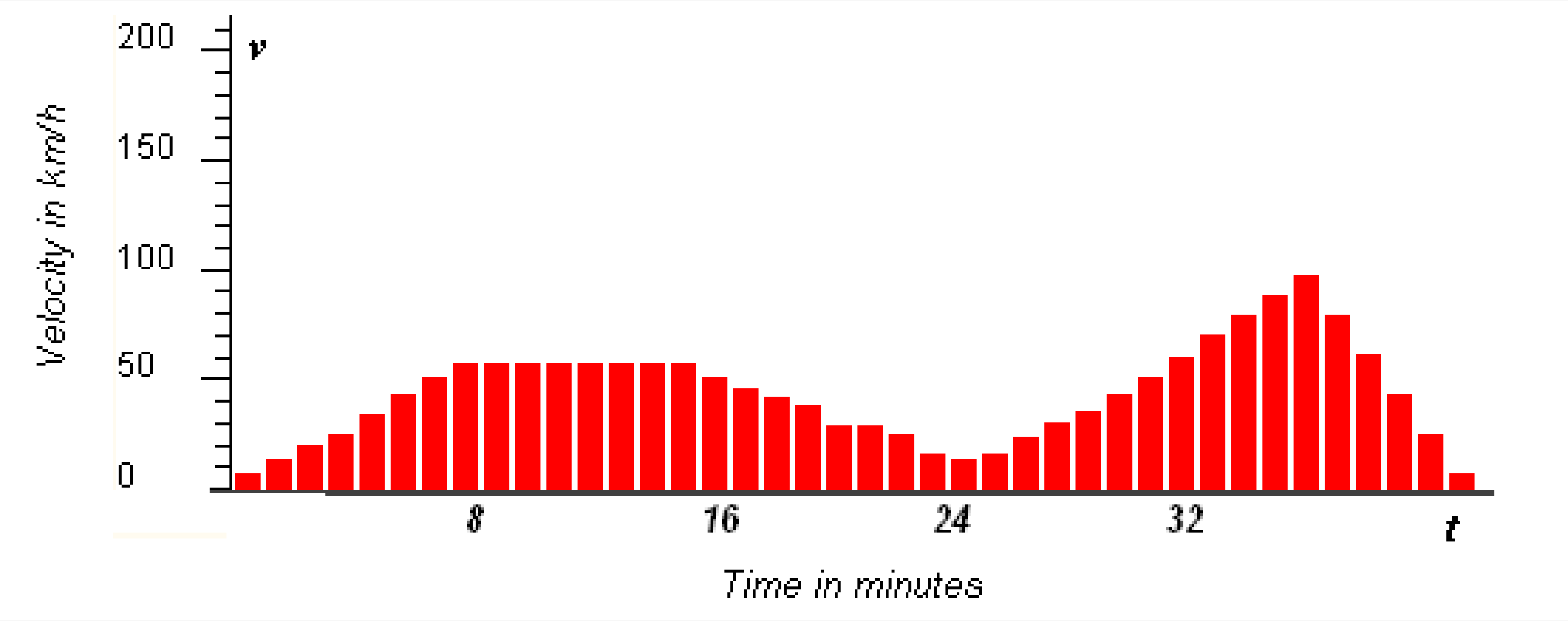
- fnInt(
- oppervlaktes grafiek
- wentelen om x-as & y-as.
- primitiveren..
- domein/bereik..
- pittig..
- veel oefenen dan lukt het wel.

Integrals

- Traditionally they are introduced in a formal way using formal notation
- A more intuitive way to approach this subject is to use the notion of accumulation
- The focus is on the concept integral function and the definite integral is seen as a partiular value of the integral function

Task 1(the train travel)

A train runs on a railroad track. Every minute, the speed is measured. The graph below shows the results of the measurements during a 40-minute drive. From the velocity- time graph it is possible to deduce how far the travel was.



Which questions would you pose in this situation?

Task 1(the train travel)

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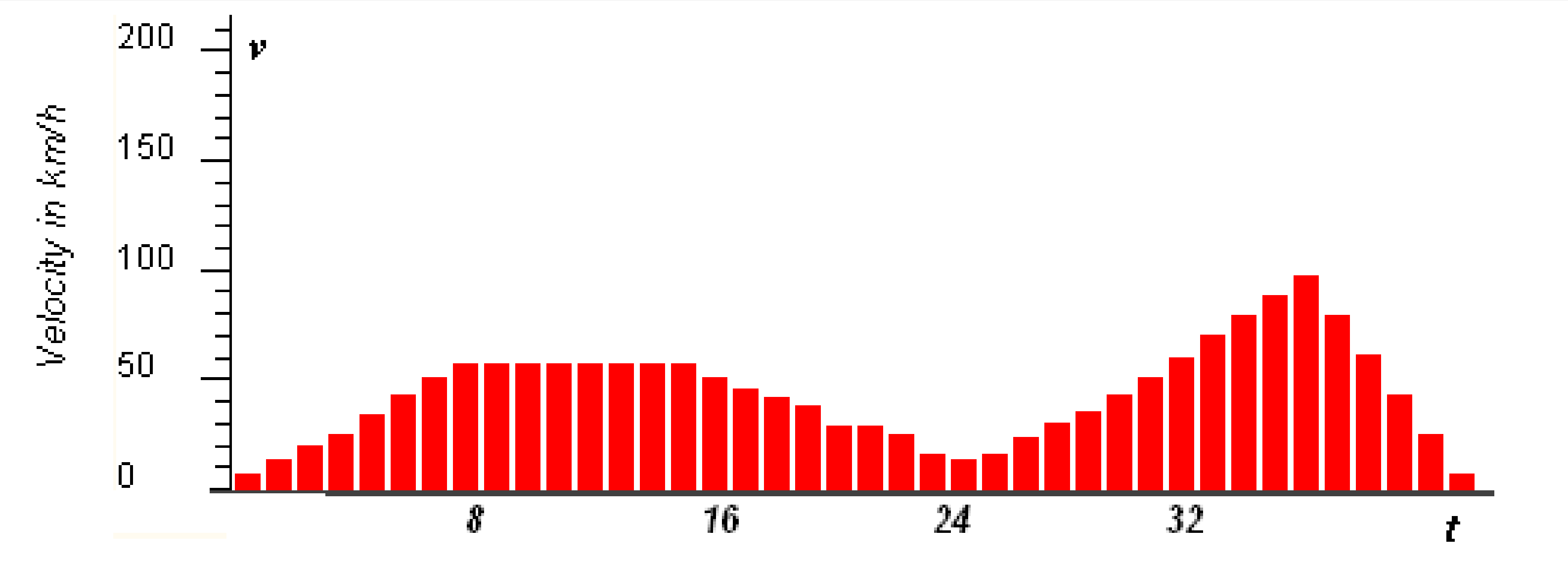


Autonomy support

Which elements are/can be included in this situation in order to create an autonomy supportive learning activity?

Structure

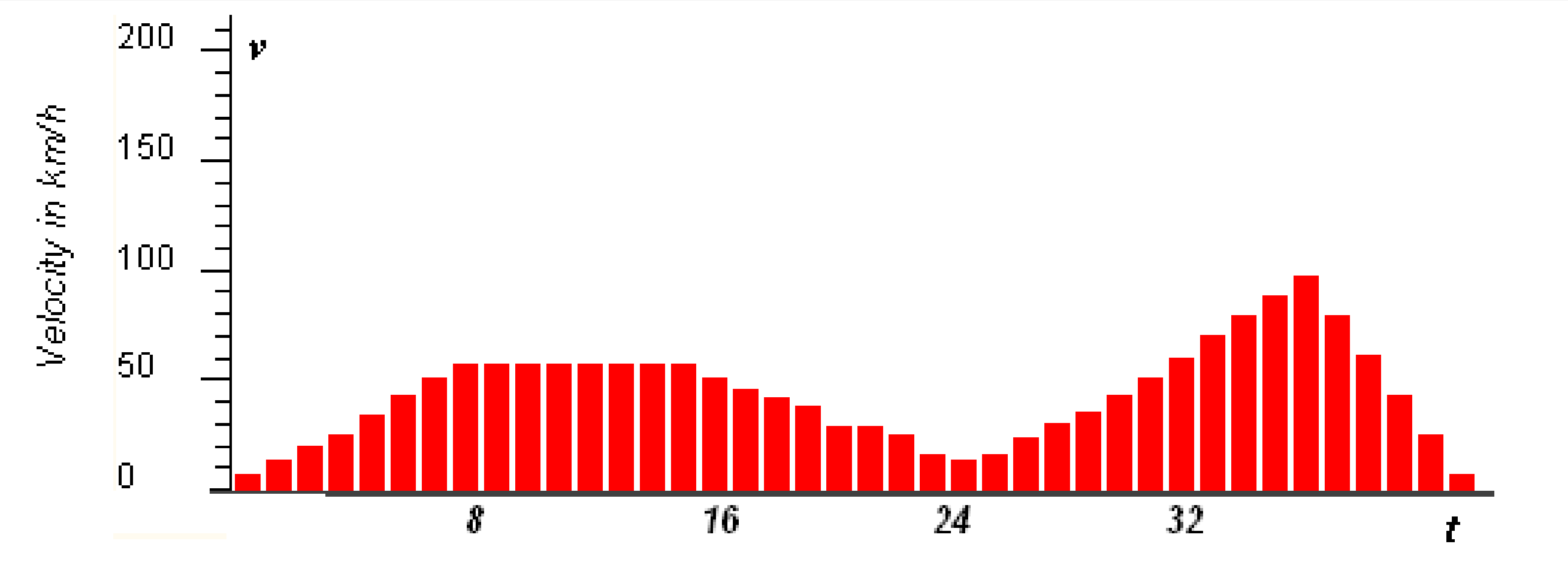
Which learning goals will you pose?

Which questions/sub-tasks could support students to achieve the learning goals?

Which feedback would you provide?

Task 1(the train travel)

A train runs on a railroad track. Every minute, the speed is measured. The graph below shows the results of the measurements during a 40-minute drive. From the velocity- time graph it is possible to deduce how far the travel was.



The issue is how much structure do students need, when solving this problem, in order that the challenging effect of the tasks remains?

Task 1(the train travel)

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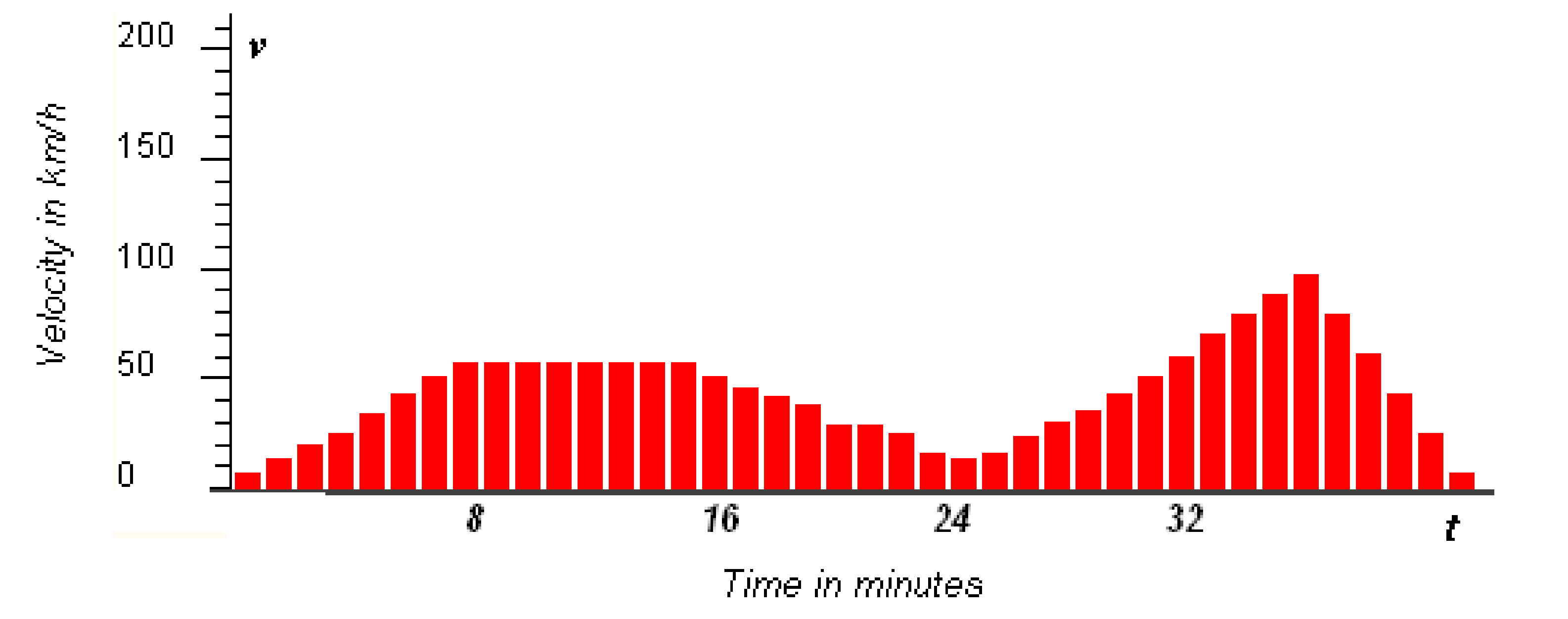

a. After how many minutes did the train travel for 15 km? Explain
b. Did the train travel more or less than 35 km in 40 minutes? Explain
c. Make a sketch of a graph that represents the traveled distance against time.
d. Explain how the questions *a* and *b* can be answered with your answer to question *c*.

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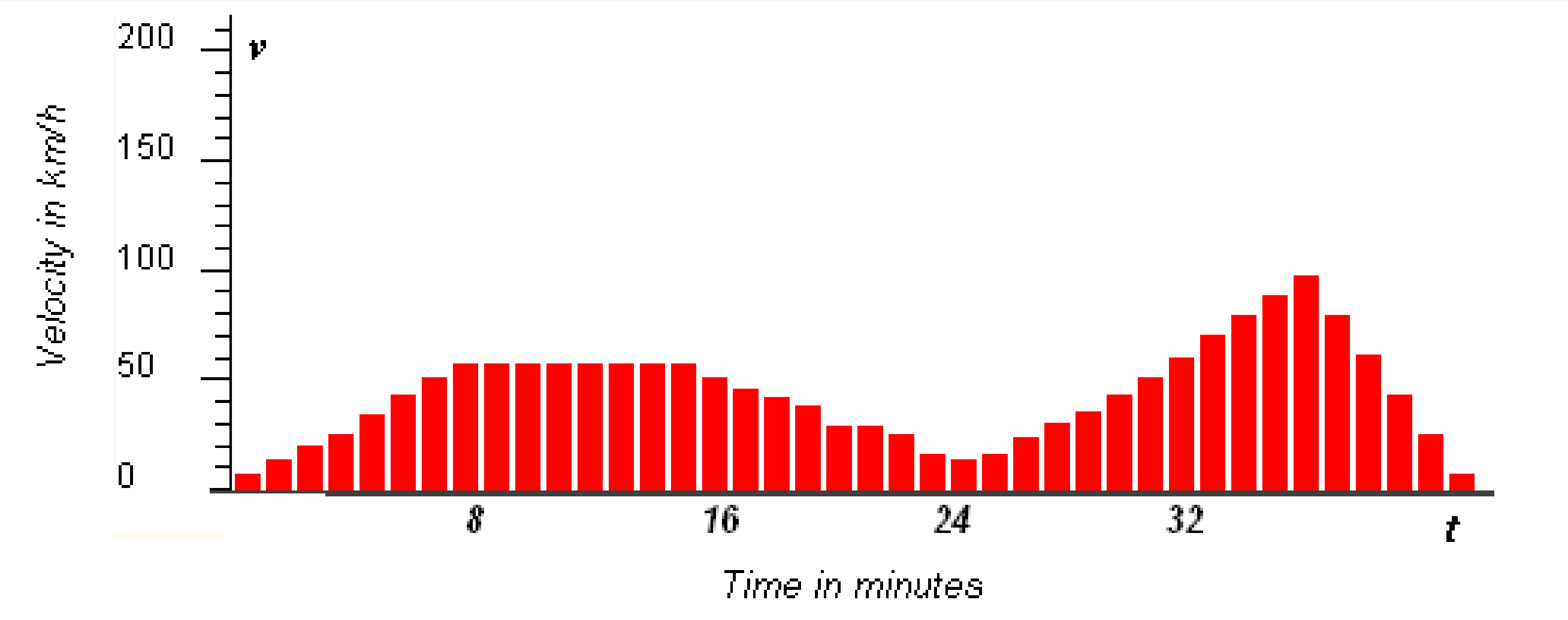

a. How far did the train travel?
b. Think about as many different (but meaningful) ways you can to answer question *a*.

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Task 1(the train travel)

Rita, 17 years old, Dutch student in 12th grade pre university education.

a. After how many minutes, did the train travel 15 km? Explain
b. Did the train travel more or less than 35 km in 40 minutes? Explain
c. Make a sketch of a graph that represents the traveled distance against time.
d. Explain how the questions *a* and *b* can be answered with your answer to question *c*.

Rita showed difficulty in connecting the task with her previous knowledge on Integral calculus; she recurred to her knowledge about distance-velocity graphs learned in sciences; the sub questions helped her to solve the task

Bram, 17 years old, Dutch student in 12th grade pre universitary education.

a. How far did the train travel?
b. Think about as many different ways (but meaningful) as you can to answer question *a*.

Brian showed difficulty to connect the task to his experiences with integral calculus. He knew how to calculate each distance and the total distance but he didn't. He doubted about his approach and he was convinced that there should be a more efficient way. Question b made no sense to him since he could not answer question a.

Task 2

Structuring and unstructuring a learning task

- Example: the parachute
  - for Group A: structure the open version
  - for Group B: “unstructure” the closed version

  In both cases the learning task should comprise some elements for autonomy support

  In both cases the learning goal is the same: use of integrals (with relation to accumulation) to find a total distance
- Discussion
- One version; students response

Task (the parachute)

A parachutist jumps from the airplane. The first five seconds she makes a free fall. Then she opens the parachute and because of that her fall velocity decreases linearly down until after 6 seconds she achieves a fall velocity of 4 meters per second. From this moment on the velocity remains constant during 70 seconds and she lands on the ground at this velocity

Task 2(the parachute)

A woman is practicing parachute jumps from a plane. In de first five seconds she falls free. Then she opens the parachute. The velocity decreased proportionally until she achieves the velocity of 4 meter per second 6 seconds after opening. From this moment on her velocity remains constant for 70 seconds. And she lands with this velocity.
Take the time in seconds with t = 0 as the moment at which she jumps from the plane.

a. During the free fall the velocity is given by the formula $v(t)$=9,8t with v in meters per second. The total falling distance can be calculated with the integral $\int_0^5 v(t)\,dt$. Calculate this distance.

b. With how many meters per second does the fall velocity decrease after opening the parachute?

c . Find the formula for v as function of t for $5 < t < 11$

d. What distance does the parachutist fall during the first 6 seconds after opening her parachute?

e. At what height did the parachutist jump from the plane?

Task a mathematics model

introduction

Dynamic proces such as train traivelling, a car drive anprocesses processen zoals een treinrit, een rijdende auto en falling situations can be descrived through a mathematica model. Which can be a tabel, a grafiek, a formula or a combination of the forms. You can use these models to solve problems or to construct new models. For instance, consider the following situation

Example Parachute jump

 A parachutist jumps from the airplane. The first five seconds she makes a free fall. Then she opens the parachute and because of this her fall velocity decrease linear down until she 6 seconds later achieves a fall velocity of 4 meter per second. From this point the velocity remains constant during 70 seconds and she lands on the ground at this velocity.

The fall process of another parachutist will be probably similar and it can be described with a mathematical model. You can also describe other processes related with velocity, as the total distance travelled by the parachutist. Often, you start with a concrete example andthe you try to find a more or less general reprsentation

In a group of three students you are going to make in this lesson a mathematical model that describes the total travelled distance of a parachute jump against time. To help you we will give a concrete example (task 1) before you go to create your own model (task 2)

TASK 1

In the example above take the time in seconds, $t = 0$ as th emoment she springs from the plane. During the free fall the velocity is given by $v(t)= 9{,}8\ t$ with v in meters per seconde.

What distance did the parachtisite travelled duting the free fall and in total?

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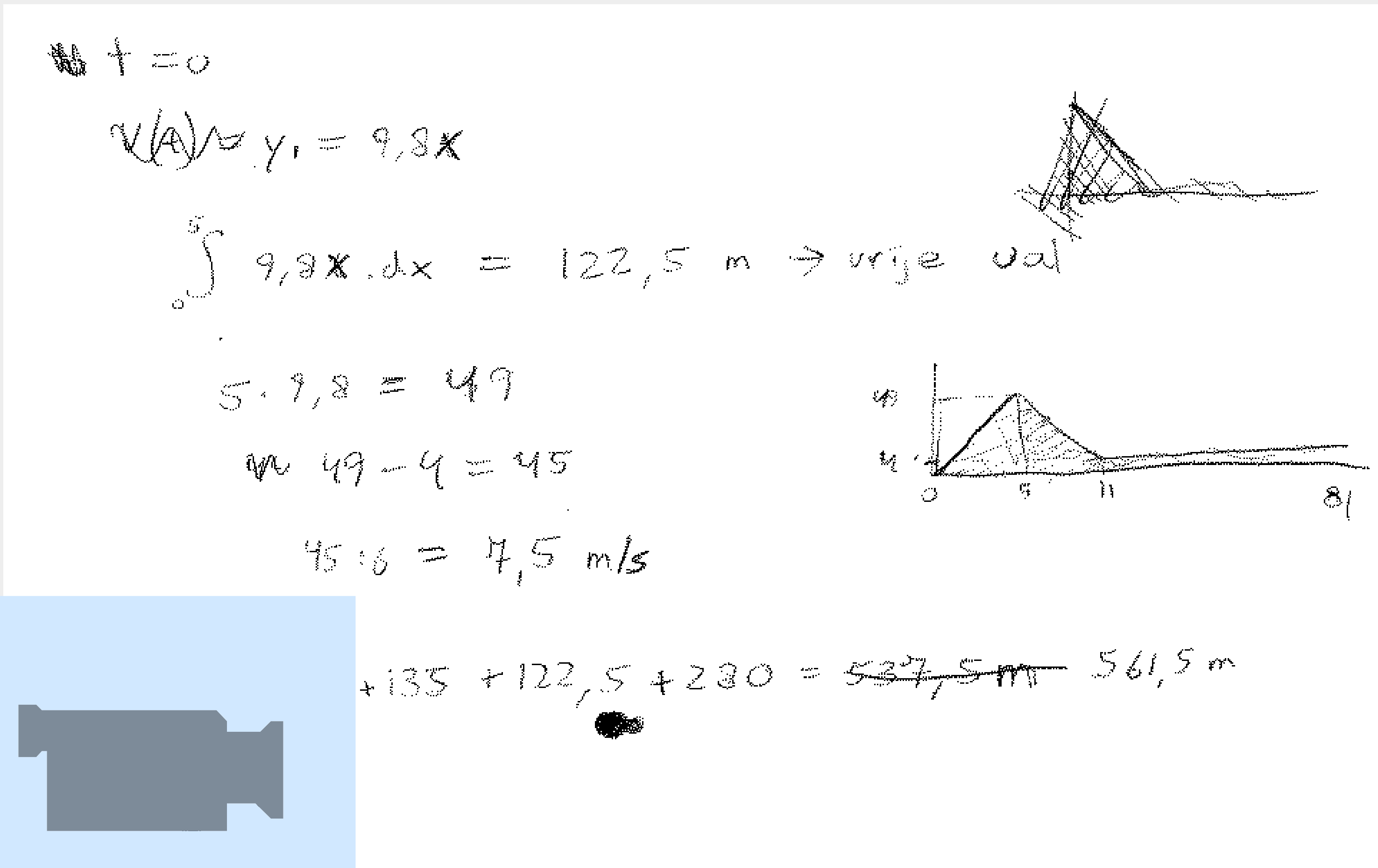
Groups of students: Bram, Sam and Jan



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Thank you

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