Local Instructional Theories and Routines of Practice: Supporting Teachers to Engage Students in Fraction-Based Algorithmic Thinking

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Purpose of Session

Share first-year findings from a larger study examining teacher practice to identify key routines of practice (Kazemi, Stipek, & Battey, 2007), and related local instructional theories (Gravemeijer & van Galen, 2003), that exemplary teachers draw upon as they engage students in fraction operation algorithmic thinking (addition and subtraction) using a guided-reinvention approach (Gravemeijer & van Galen, 2003)
Learning to Operate with Fractions vs. Engaging Students in Algorithmic Thinking

- Common Approach to Learning to Operate with Fractions:
  - Work with Visual Models, Explain and Model Algorithm, Practice Algorithm Symbolically/Trying to Concretize Abstract Knowledge
    Freudenthal, 1983

- Guided-Reinvention Approach to Algorithmic Thinking:
  - Students have responsibility to engage in the mathematical development of ideas—algorithmic thinking is a form of mathematical activity
    Gravemeijer & van Galen, 2003
  - Use contexts and models that draw out informal solutions that can then serve as a bridge to formal reasoning
    Fosnot & Dolk, 2001
  - Contexts, models and symbolism serve as tools for thinking and through this students develop number sense/operation sense that lead to articulation of meaningful algorithms

From Theory Into Practice

Algorithmic Thinking using Guided Reinvention:
“Guided reinvention asks for topic-specific instructional theories. To be able to organize learning processes, teachers need ‘local instructional theories’ for each topic.”
Gravemeijer & van Galen, 2003

Routines of Practice:
“We propose that the field needs to identify routines of practice...core activities (within each mathematical domain and at appropriate grade levels) that could and should occur regularly in the teaching of mathematics.”
Franke, Kazemi & Battey, 2007
Context: 4 Project Goals

1. Understand and document routines of practice (and related instructional theories) that exemplary teachers use as they engage students in algorithmic thinking for fractional operations.
2. Use outcomes from Goal One to develop a prototypical model of core routines of practice composed from exemplary teachers that support students as they engage in algorithmic thinking for fraction operations.
3. Design, pilot and study the usability of the prototypical model of core routines of practice as a professional development tool with typical teachers as they engage students in algorithmic thinking for fraction operations.
4. Identify the core routines of practice from Goal Three that are shown to be productive with typical teachers and explore ways of disseminating them at a larger scale.

Phase 1 of Research

- Exemplary Practice: Study practice of 4 exemplary teachers using the CMP2 Bits and Pieces II: Using Fraction Operations (Lappan et al.)
- Collaborative study of teacher practice: Observation, video, interviews, Summer meeting in Year 1 and Year 2
- Immediate Goal: Unpack each teacher’s practice and their local instructional theories for engaging students in algorithmic thinking.
- Long-term Goals: Routines of Practice & PD materials developed in Year 3 to be piloted and revised in Phase 2 or Years 4 and 5.
Research Focus

- Teachers are implementing a curricular unit developed using design research—CMP II Bits and Pieces II: Using Fraction Operations  
  Lappan, et.al, 2006
- Don’t assume the instructional sequence “just works”. Focus is on what are the content-specific practices that exemplary teachers use to support students as they engage in algorithmic thinking.
- Unpack exemplary teacher practice in this domain

Analytic Framework

- Content: 
  - Mathematical Development
  - How Representations are Developed and Used
  - Purposefulness
  - Discursive Considerations for Positioning the Learner
- Content: Greeno’s (1991) number sense as a conceptual domain
- Generic Work on Practice: 
  - Mathematical Reasoning: (Ball & Bass, 2003)
  - Sociomathematical Norms (Kazemi & Stipek, 2001)
Prior to Fraction Operation Unit

- Part-whole, operator, measure, indicated division (rate/ratio not addressed)
- Contexts, physical models, drawings, patterns,
- Linear strip or number line models, area models, grid models,
- Equivalence (including mixed/improper fractions)
- Fraction-decimal-percent relationships
- Benchmarks

The Land Problem Part A
Implementing the Land Problem Part A

- **Teacher One: Jim Mamer**
  - Mamer Day 5: Tanner on Bouck (13:00-16:00)
  - Mamer Day 5: Maddie on Burg (27:10-29:10)

- **Teacher Two: Nancy Wood**
  - Wood Day 4: Maps 35:50 “Let’s take a look at these [maps] and look at their numbers. They have not written any strategies on here, so we will have to think about strategies after the break. Take a look at the fractions they have written, see if there are any you want to discuss. See if you had a different answer, that they turned out to be equivalent. Look carefully and see if you agree.
  - Wood Day 5: Autumn and Alexis on Stewart (25:00-29:50)

Reactions to Mamer and Wood

- **General Impressions?**
- **Proposed Framework:**
  - Mathematical Development
  - How Representations are Developed and Used
  - Purposefulness
  - Discursive Considerations for Positioning the Learner
- **Focus on Teacher Practice**
- **What mathematical ideas emerged that could support the development of an +/- algorithm?**
Mamer’s Trajectory in BPII Unit

Investigation 1: Estimation

- Purposefully Altered “Getting Close” Game and shifted focus to establishing benchmarks based on the unit conveyed in denominator
- Using unit indicated by denominator to focus on “closer to 0, ½, 1 whole” and “how far”
- “How far” focus was purposeful. It was not the focus of the text.
- Purposefully altered the design of the “Estimating Sums” problem.

Mr. Mamer: Estimation

Day 2: Determining How Far?

- Where would 4/9 be on the number line?
- If you are estimating, is it closer to 0, ½ or 1 whole?
- How far is 4/9 from 0?
- How far is 4/9 from 1 whole?
- What does the 9 in 4/9 tell you?
- What does the 4 in 4/9 tell you?
Mr. Mamer: Estimation

- Estimation is a way to find an approximate but reasonable solution
  - Day 4 Problem 1.2 C1: Using “how far” reasoning and benchmark reasoning (Day 4, 7:15-10:00)
- Estimation is a way to assess the reasonableness of an algorithmic approach
  - Day 3 Problem 1.2 B2: Is the computation $\frac{7}{12} + \frac{5}{8} = \frac{12}{20}$ reasonable?
  - “We are hoping that your estimate will show that $[\frac{7}{12} + \frac{5}{8} \neq \frac{12}{20}]$. When you did your estimate, you should have seen that $\frac{12}{20}$ is not close to what the real answer should be.”

Mr. Mamer Land Problem B, C, D

What stands out as key aspects of Mr. Mamer’s practice that supports the emergence of the algorithm for adding and subtracting fractions?

Problem B:
- Fuentes + Theule (Day 6, 0:00-5:05)

Problem C:
- Making 1 ½ Sections (Day 6, 18:00-20:00)

Problem D:
- Bouck + Lapp = Foley (Day 6, 25:30-28:32)
Emerging Number Sentences

- **Fuentes + Theule:**
  - $\frac{1}{16} + \frac{3}{16} = \frac{4}{16}$
  - $\frac{6}{32} + \frac{2}{32} = \frac{8}{32}$
  - $0.5 + \frac{1}{5} = \frac{2}{32}$
  - $\frac{1}{16} + \frac{6}{32} = ?$

- **Making 1 1/2 sections:**
  - $1 + \frac{3}{16} + \frac{1}{16} + \frac{4}{16} = \frac{18}{16}$
  - $\frac{4}{4} + \frac{3}{16} + \frac{5}{16} = \frac{18}{16}$
  - $\frac{16}{16} + \frac{3}{16} + \frac{5}{16} = \frac{24}{16}$

- **Bouck + Lapp = Foley**
  - $\frac{1}{4} + \frac{1}{16} = \frac{5}{16}$
  - $\frac{1}{4} + \frac{1}{16} = ?$

Developing Written Record

![Image of a written calculation]

Since lessons:

- $\frac{2}{5} + \frac{3}{10} + \frac{9}{10} = \frac{7}{5}$
- $\frac{1}{3} + \frac{1}{5} + \frac{1}{1} = \frac{5}{5}$
- $\frac{1}{5} + \frac{1}{10} = \frac{1}{10}$
Articulating Strategies and Examining Each Others Written Record

Mr. Mamer: Articulating an Algorithm

- Problem 2.4 in the curriculum is designed for ask students formally articulate an algorithm.
- Mr. Mamer has drawn it out and it is on the table across Days 6-11 before Mr. Mamer asks them to formally write an algorithm on Day 12.
Emergence of an Algorithm

- What stands out as key aspects of Mr. Mamer’s practice that supports the emergence of the algorithm for adding and subtracting fractions AND engaging students in algorithmic thinking?
  - Mathematical Development
  - How Representations are Developed and Used
  - Purposefulness
  - Discursive Considerations for Positioning the Learner

Mr. Mamer: Key Elements of Practice

- Estimation as a Tool
- Equivalence as a Tool
- Diagrams/Models as a Tool
- Symbolism as a Tool

Movement back and forth between equivalence, representation and symbolism.

Estimation is not strongly used once Land Problem started, but the “how far” reasoning appears important initial development of number sense.
Mrs. Wood’s Trajectory in BPII Unit

Investigation One Estimation
- Used Getting Close game as outlined to fold back to previous ways to think about fractions as magnitudes
- Uses game to think about how to reason when estimating a sum rather than benchmarking a single number
- 1.1 led to “close to a benchmark”, “a little more or little less a benchmark” and “how far from a benchmark”
- Focus on flexibility while estimating: which strategy depends on numbers. (Day 1, 45:49)
Estimating Sums Strategies

Problem 1.2: Did all of 1.2 but didn’t jump the gun on the algorithm.

Problem 1.2B: Problematizing with a push for using estimation as a tool (Day 2, 58:40-1:01:45)

Using Estimation as a Tool: Hannah’s Computation Error for 1.2D3 (Day 3, 30:10-33:00)
Mrs. Wood: Land Problem and Langston’s

- In earlier intro clips “having the same type of pieces is needed when trying to say what fraction of land a person has” (Autumn 2/16 + 1/32)
- Langston’s Flowers (Day 6, 18:45-20:18)
- Pr. 2.1C. Begins by asking: “What is the most important thing you need to do to combine landowners?”

Mrs. Wood: +/- Mixed Numbers

- Spice Problem B1: Looking at AZ’s work and why she uses 12ths. (Day 7, 9:40-16:00)
- Spice Problem B1: Ibrahim’s Chunking Strategy for Adding Fractions (Day 7, 21:00)
- 3 Strategies for Subtraction Emerge
Using Estimation/Articulating Strategies

Strategies we use to Add and Subtract Fractions

- Estimate
- Find a common denominator
  Product of denominators
  If one is a factor - Match
- Add or subtract numerators
- Simplify

Using Estimation

<table>
<thead>
<tr>
<th>Group 1</th>
<th>Group 2</th>
<th>Group 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1\frac{1}{3} - 2\frac{2}{3}$</td>
<td>$6\frac{3}{4} - 3\frac{7}{8}$</td>
<td>$3\frac{1}{4} - 1\frac{5}{6}$</td>
</tr>
</tbody>
</table>

$1\frac{1}{3} = \frac{4}{3}$

$6\frac{3}{4} = 6\frac{3}{4} = \frac{5}{4}$

$-3\frac{3}{8} = \frac{5}{8}$

$2\frac{7}{8}$
Mrs. Wood: Writing an Algorithm

- Estimation
  - Day 9 at the end of 2.3 but before 2.4 (See next slide)
  - Day 10 when doing 2.4 (See slide after that)

- Writing a Common Algorithm using 10 11/16 - 3 7/8
  (Day 11, 12:30-13:55, scan, 27:25-29:20, next slide)

Writing a Common Class Algorithm

- Algorithm for Adding and Subtracting Fractions
  1. Estimate
  2. Find a common denominator. Comm if needed.
  3. Add or subtract numerators
     - If needed:
       - Improper fractions
       - Mixed numbers
       - Subtract subtraction subtraction
  4. Simplify to lowest terms
  5. Check to see if reasonable
     - Compare to estimate
Emergence of Algorithm

- What stands out as key aspects of Mrs. Wood’s practice that supports the emergence of the algorithm for adding and subtracting fractions AND engaging students in algorithmic thinking?
  - Mathematical Development
  - How Representations are Developed and Used
  - Purposefulness
  - Discursive Considerations for Positioning the Learner

Mrs. Wood: Key Elements of Practice

- Estimation as a Tool
- Equivalence as a Tool
- Diagrams/Models as a Tool
- Symbolism as a Tool

Movement back and forth between estimation, representation and symbolism.

They use equivalence but it is more implicit or “taken as shared” until the very end when they write the common algorithm.
Mrs. Wood

Looking Forward: Routines of Practice

Proposed Focus:
- Purposeful Development Of:
  - Equivalence as a Tool (including “how far” reasoning)
  - Estimation as a Tool
  - Use of Visual Model as a Tool
  - Symbolism as a Tool to Represent and Communicate Mathematical Ideas (Written Record)
- Highlight the movement between/among tools as highlighted in teachers’ practice
- Role of Language
Mathematical Reasoning: Ball & Bass

Offered 2 key aspects when developing reasoning of justification in students: body of public knowledge and language

- “Mathematical language is the foundation of mathematical reasoning.”
- Mathematical Language: Symbols, terms, and other representations and their definitions-and rules of logic and syntax for their meaningful use in formulating claims and the results of relationships used to justify them.

Sociomathematical Norms: Kazemi & Stipek

- An explanation consists of a mathematical argument, not simply a procedural description or summary;
- Mathematical thinking involves understanding relations among multiple strategies;
- Errors provide opportunities to reconceptualize a problem, explore contradictions in solutions, or pursue alternative strategies; and
- Collaborative work involves individual accountability and reaching consensus through mathematical argumentation.
An algorithm is nice, but it is not the only goal. We want to develop many strategies...and we want students to develop the ability to look at numbers and decide what is an efficient approach.
Final Thoughts

Questions?

Comments?

Thank You!

THE

END