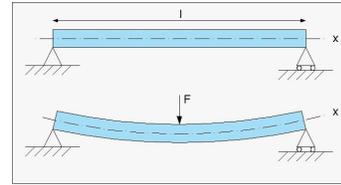


# Proportionality

## A. Bending of a beam

A beam with length  $l$ , width  $w$  and height  $h$  (all in mm) is supported at both ends.

Due to a force  $F$  (in N), the beam will bend.



The flexion  $f$  (in mm) in the middle of the beam can be found using this complex looking formula:

$$f = \frac{c \cdot F \cdot l^3}{E \cdot w \cdot h^3} \cdot 9.81$$

$E$  is Young's modulus (or: modulus of elasticity), measured in  $\text{N}/\text{mm}^2$ .

Young's modulus is a measure of the stiffness of an elastic material and is a quantity used to characterize materials.

A1. A higher value of  $E$  means that the material is more/less stiff. What is the correct word here: 'more' or 'less'? Explain!

A2. How does doubling the length  $l$  of the beam affect the flexion  $f$ ?  
Same question for doubling width  $w$ , height  $h$  and force  $F$

A3. Why do joists have a shape as shown in the picture?

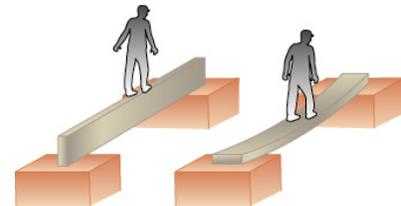


The cross section of a supporting beam has a fixed area of  $100 \text{ cm}^2$ .

Three different shapes with that area ( $w \times h$ ):  $20 \times 5$ ;  $10 \times 10$ ;  $5 \times 20$

A4. How do the flexions of the three beams compare?

A5. Use the formula for the flexion to explain what you see in this picture



## B. Bicycles



Find the relationship between  $b$ ,  $f$  and the entries  $d$  in the table.  $f$  is the number of teeth on the front sprocket;  $b$  is the number of teeth on the backwheel sprocket;  $d$  is the distance (in cm) traveled by a bicycle for one rotation of the pedals. The values of  $d$  are rounded to the nearest integer.

*Hint: look for patterns in the (horizontal) rows and in the (vertical) columns or: where does the number 431 appear in the table... why there?*

	$f$														
	26	28	30	32	34	36	38	40	42	44	46	48	50	52	54
12	467	502	538	574	610	646	682	718	754	790	826	861	897	933	969
14	400	431	461	492	523	554	585	615	646	677	699	739	769	800	830
16	350	376	403	431	458	484	511	538	565	593	619	646	673	700	727
18	311	335	359	383	407	431	455	479	502	526	550	574	598	622	646
20	280	301	323	345	366	388	409	431	452	474	495	517	538	560	581
22	254	274	294	313	333	353	372	392	411	431	451	470	490	509	528
24	233	251	269	287	305	323	333	359	377	395	413	431	449	467	484
26	215	231	249	265	282	298	315	331	348	365	381	398	414	431	447
28	199	215	230	246	262	277	292	308	323	338	354	369	384	400	415