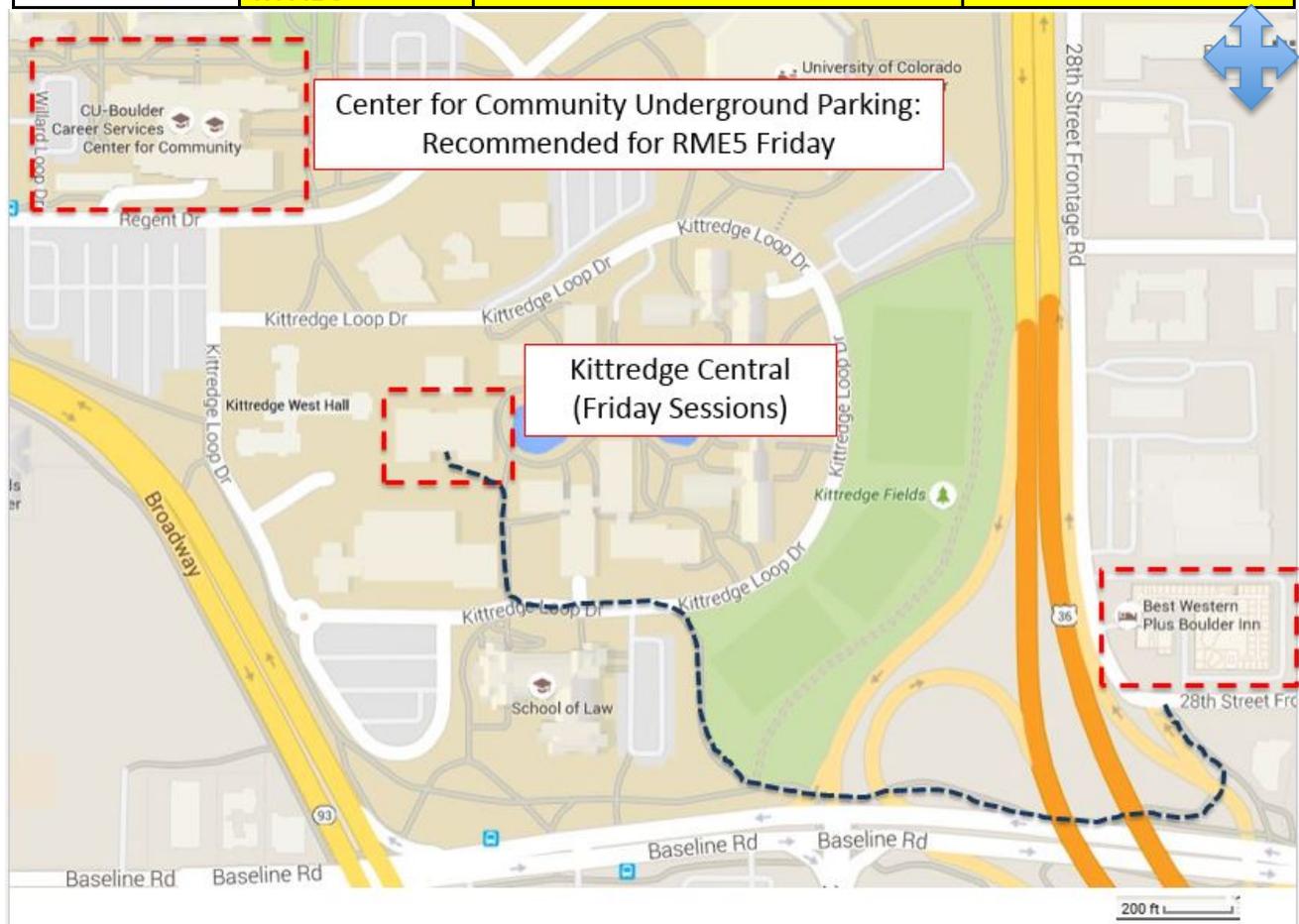# 5th International Realistic Mathematics Education Conference
Using Realistic Contexts and Emergent Models to Develop Mathematical Reasoning
University of Colorado Boulder
September 18 – 20, 2015

Schedule and Program

Friday, September 18th
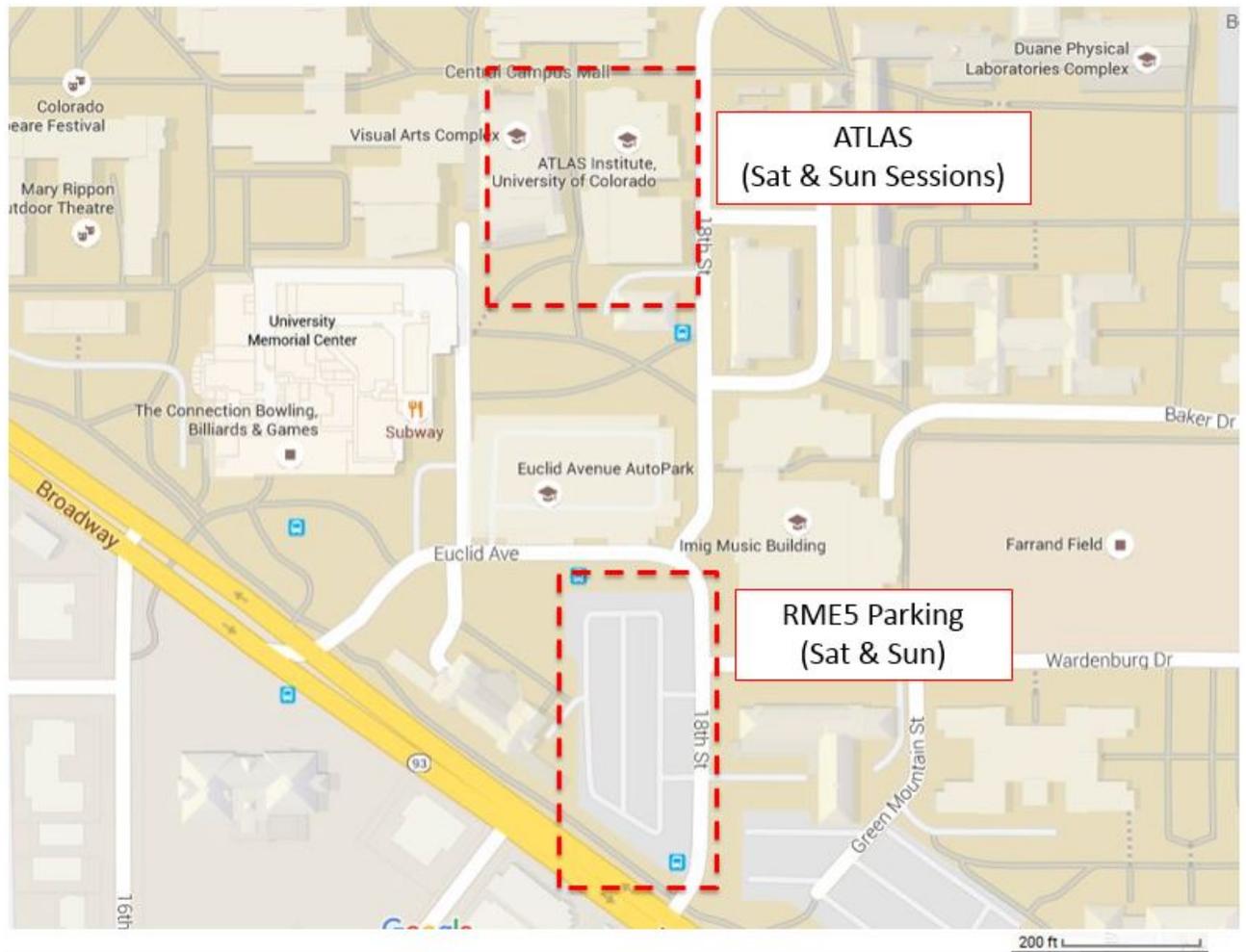*Onsite Registration available in the Kittredge Central Lobby from 8 – 10 am*

08:00 – 08:45	Full breakfast **Kittredge Central Lobby**			
08:45 – 09:00	Welcome and opening **Kittredge Central N114BC**			Henk van der Kooij David Webb
09:00 – 10:30	Plenary 1 **Kittredge Central N114 BC**	**An Orientation to Realistic Mathematics Education**		Johnson, Peck, et al
10:30 – 11:00	Break			
11:00 – 12:30 Parallel Sessions	Interactive 1	Irma Vazquez & Jacqueline Sack **Kitt Cen N114A**	Mieke Abels & Martin Kindt **Kitt Cen N114B**	Gloriana Gonzalez **Kitt Cen N114C**	Sonia Palha **Kitt Cen N114D**
12:30 – 01:30	Lunch **Kittredge Multipurpose Room**			
01:30 – 03:00 Parallel Sessions	Interactive 2	Kara Imm (modeling) **Kitt Cen N114A**	Frans van Galen **Kitt Cen N114B**	Heather Johnson **Kitt Cen N114C**	Jacqueline Sack & Judith Quander **Kitt Cen N114D**
03:00 – 03:30	Break			
03:30 – 05:00	Plenary 2 **Kittredge Central N114BC**	**A personal journey in the (re)design of teaching and learning math**		Henk van der Kooij

Center for Community Underground Parking: Recommended for RME5 Friday

Kittredge Central (Friday Sessions)

Saturday, September 19th

Onsite Registration available in the ATLAS Lobby from 8 – 9:30 am

08:00 – 8:45	Full breakfast **ATLAS Building West Walkway**				
8:45 – 10:00	Plenary 3 **ATLAS 100**	**Beyond flatland in primary school mathematics education in the Netherlands**		Marja van den Heuvel-Panhuizen	
10:00 – 10:30	Break				
10:30 – 12:00 Parallel Sessions	Interactive 3	Frank Eade **ATLAS 1B31**	Anna deJarnette **ATLAS 1B29**	Takehiro Tsubokawa, Yuko Ichikawa & Mari Morimoto **ATLAS 1B25**	Fred Peck **ATLAS 104**
12:00 – 01:00	Lunch **ATLAS Building West Walkway**				
01:00 – 02:30 Parallel Sessions	Interactive 4	Kara Imm (number) **ATLAS 1B25**	Brent Milne & Terry Wyberg **ATLAS 104**	Kevin Reins **ATLAS 1B31**	Daniel Reinholz & Dmitri Dounas-Frazer **ATLAS 1B29**
02:30 – 2:45	Break				
2:45 – 4:00	Plenary 4 **ATLAS 100**	**Post-secondary mathematics: keeping it real**		Eric Stade	

Sunday, September 20th

Time					
8:15 - 9:00	Full breakfast **ATLAS Building West Walkway**				
9:00 - 10:15	Plenary 5 **ATLAS 100**	**Fi-gured Algebra**			Martin Kindt
10:15 - 10:45	Break				
10:45 - 12:15 Parallel Sessions	Interactive 6	Sherri Farmer **ATLAS 104**	Pamela Harris **ATLAS 1B29**	Elizabeth Kimber & Anna Baker **ATLAS 1B25**	Peter Boon **ATLAS Tech**
12:15 - 1:30	Lunch **ATLAS Building West Walkway**				
1:30 – 3:00 Parallel Sessions	Interactive 7	Marja van den Heuvel-Panhuizen & Mieke Abels **ATLAS Tech**	Amanda McGarry **ATLAS 104**	Riana Adams **ATLAS 1B31**	Maximillian Gerrick **ATLAS 1B25**
3:00 - 3:15	Break				
3:15 - 4:00	**ATLAS 100**	**The Challenge and Promise of RME**			David Webb

Session Titles and Abstracts

Sessions are organized below according to the surname of the lead speaker.

Abstracts for Plenary Sessions

Beyond flatland in primary school mathematics education in the Netherlands
Marja van den Heuvel-Panhuizen, Freudenthal Institute, University of Utrecht
The Netherlands has a strong tradition in teaching mathematics following the principles of Realistic Mathematics Education (RME). Characteristic for this approach to mathematics education is that it starts with offering students problems in meaningful situations, of which the contexts can gradually evolve into didactical models that can be used to solve a broader scope of problems, and, through progressive schematization, eventually ends up in understanding mathematics at a more formal level. This process of mathematization can be distinguished in horizontal (from the world to the mathematics) and vertical mathematization (within the mathematics). However, in RME as it is conceptualized in primary school mathematics textbooks (and consequently in classroom practice), there is not much attention for vertical mathematization, which actually is restricted to carrying out plain operations with numbers. This results in a rather 'flat' primary school curriculum which does not give students a good basis for developing higher-order thinking in mathematics. Therefore, new research-based design is necessary to make the primary school mathematics curriculum more mathematical.

An Orientation to Realistic Mathematics Education
Raymond Johnson, FIUS, University of Colorado Boulder
Fred Peck, FIUS, University of Montana
William Campbell, Ryan Grover, Susan Miller, Ashley Scroggins & David C. Webb,
FIUS, University of Colorado Boulder
Realistic Mathematics Education is distinguished by a rich history and a number of core principles and design heuristics that distinguish it from other approaches to mathematics education that use context, manipulatives, and real-world phenomena. This opening plenary will orient participants to the conference by surveying the primary theories and concepts that guide RME and detailing some of the exemplary work representing RME research and design.

Fi-gured Algebra
Martin Kindt, Freudenthal Institute, University of Utrecht
The main image of 'algebra' at school is manipulating with symbols and formal expressions. But algebra is much older than 'algebra'. More than 3500 years ago the Babylonian mathematicians could solve every second-degree equation, without using symbols. They used geometric terms; width and length for unknowns, rectangle for product. The Pythagoreans used patterns of dots to describe special sequences of numbers like squares and triangular numbers. This presentation will propagate a visual approach of algebra and also discuss some challenging problems. These type of problems provoke a productive use of algebraic rules and promote an understanding for algebra.

A personal journey in the (re)design of teaching and learning math
Henk van der Kooij, Freudenthal Institute, University of Utrecht
Working as a high school teacher since 1975, I was asked in 1987 to join a small team to re-design a Dutch senior high school program (grades 10 and 11). That was my start of working for the Freudenthal Institute. Several design projects followed until 2010, among them the NSF-funded high school reform project AR!SE (Solomon Garfunkel, COMAP), TechMap (COMAP), TWIN (a Dutch reform project for vocational education in engineering) and other more small-scale projects in the Netherlands and the USA. I want to look back with you at the most important aspects of all that work over the past 30 years. How my ideas about "mathematics for all" changed over time. How "theoretical learning trajectories" (the basis for the design of teaching and learning materials) sometimes turned out in practice as eye openers for alternative students' thinking. Learning mathematics for students is not learning to imitate your teacher's professional math knowledge. In my opinion, every student deserves enough space to develop his/her own construct of math and opportunities to discuss personal ideas with fellow students and the teacher to improve his/her mathematical conceptual knowledge and (more algorithmic) skills. Be prepared to be challenged to solve intriguing problems from several curriculum projects, because at least 40 minutes of my 90 minutes presentation will be used to make you think about and work on problems I will present!

Post-secondary mathematics: Keeping it real
Eric Stade, University of Colorado Boulder
When real-world scenarios are introduced at the very beginning of an undergraduate mathematics course, instead of somewhere in the Exercises for the Chapter 5 (for example), great things happen. These scenarios become not (just) applications of the mathematics, but contexts

for the development of the mathematics. That is, the scenarios serve the mathematics, instead of (or in addition to) the other way around. Such a contextual approach allows for development of material in a way that's completely faithful to all of the beauty and elegance of mathematics as a "pure" discipline, but is just as faithful to the utility and power of mathematics as an "applied" discipline. We explore all of this primarily through the lens of a contextual first semester calculus course, but touch, along the way, on the relevance of these ideas to other undergraduate courses, ranging from Math for Elementary Teachers to Fourier Analysis.

The Challenge and Promise of RME
David C. Webb, FIUS, University of Colorado Boulder
The evidence is quite clear. Research points to improved access and equity in the learning of mathematics when students are engaged in activities that are more relevant and authentic, when they have opportunities to propose and discuss strategies and representations with peers, and when the teacher develops a classroom in which sense making and problem solving are valued. Yet, in spite of the promise of improved engagement and learning teacher implementation of Realistic Mathematics Education is confronted by several challenges. We will explore these challenges and discus recommendations for supporting productive implementation of RME.

Abstracts for Breakout Sessions

What to do with fractions?
Mieke Abels & Martin Kindt, Freudenthal Institute, University of Utrecht
The way of teaching fractions in primary schools nowadays differs strongly from the teaching of some decades before. Now there is much more attention for the use of different visual models and fraction operations in meaningful contexts. Rules are not longer drilled endlessly. In secondary schools teachers often expect that their students have enough skills to operate with fractions in a formal way, but it often happens to be a 'miscalculation'. Particularly when algebraic fractions appear many things are going wrong. How to repair these failures? In secondary education there should be paid more attention to an insightful approach of, and challenging exercises with fractions. This may lead to generalization and formalization and it will be a base to work with algebraic fractions. At the Freudenthal Institute we developed not only some units in this spirit, but also a series of activities in the DME project (Digital Math Environment). In our presentation we will show examples of this work.

Indigenous RME Activities for Mathematical Literacy Grade 10 – 12
Riana Adams, Uplands College, Mpumalanga, South Africa
I come from an inclusive, co-ed, multi-national school in South Africa. I will take you into the minds of our learners who come to us from enormously diverse backgrounds and with a large range of challenges. My workshop consists of 10 Space, Shape and Measurement - adapted indigenous activities that will engage you in realistic mathematical scenarios. These IEP compliant activities are designed to promote a love for learning and boost confidence in the real world of numbers.

Designing RME activities for tablet devices
Peter Boon, Freudenthal Institute, University of Utrecht
In this working group we use the DME (Digital Mathematics Environment) of the Freudenthal Institute to design interactive activities for students that can be used on ipads and other tablets. Participants of this working group will get the opportunity to design their own web-apps that will be available on the Internet immediately. The DME offers a powerful authoring tool to design both explorative activities and exercises that give hints and immediate feedback. Moreover, other educational resources available on the Internet, for example the PhET Sims developed at CU-Boulder can be embedded in the DME web-apps.

Students' Use of a Computer-Programming Environment as a Realistic Context for Learning Algebra
Anna F. DeJarnette, University of Cincinnati
I will present on an after–school computer–programming club designed to examine how students use concepts from algebra when solving programming tasks. Students used Scratch to complete a sequence of tasks related to designing and beating a car–racing game. I will present examples of the tasks and of students' discussions and solutions, as a basis for considering how programming tasks can serve as realistic contexts for students to make sense of mathematics.

Introducing Realistic Mathematics Education in the Cayman Islands
Frank Eade, Cayman Island Government
The presentation will provide an account of the implementation of Mathematics in Context with low and middle attaining students in grade 6. We will examine the changes in behaviors and outcomes for the students and the support provided for the teachers to enable positive change. Although experienced high school teachers' practices are not easy to influence we will argue that with sustained and appropriate support and a very well researched scheme it is entirely possible to have strong positive impact on both practices and beliefs

Using Pictures to Support Student's Developing Understanding of Multiplication
Sherri Farmer, Purdue University
Visual media is a common mechanism for engaging and communicating information with students. This study describes students' mathematizing their world while developing unique mathematical insights using photographs as mathematical referents. Using RME foundations and Presmeg's (2006) method of semiotic "chaining," this study discusses connections between media and student's emergent mathematical ideas. It further demonstrates use of photographs as a referent object representing mathematical ideas, such as multiplication, can support the growth of children's mathematical understanding.

Designing for Realistic Mathematics Education: Learning about graphs as an example
Frans van Galen, Freudenthal Institute, Utrecht University, the Netherlands
'Guided reinvention' is a core concept in Realistic Mathematics Education. In my presentation I want to address the question what it means if we say that children have to reinvent graphs and graphing. Textbooks for mathematics tend to focus on the interpretation of graphs, not on the construction of graphs, which does not help students to understand the basic principles underlying graphs. My examples of alternative learning trajectories will come from grade 4 to 6.

Change of Representations as a Design Principle for Initiating Learning Processes for Quadratic Functions
Maximilian Gerick, TU Dortmund University, Institute for Development and Research in Mathematics Education
For a suitable development of the concept quadratic functions students must establish viable connections between graphical and symbolic representations. They often use graphical representations geometrically instead of functionally which can lead to unviable connections. Based on multiple cycles of design experiments an iterative development of a learning and teaching arrangement will be presented that helps students to use graphical representations functionally when linking them with symbolic ones.

Teachers' Understandings of Realistic Contexts for Capitalizing on Their Students' Prior Knowledge
Gloriana González, University of Illinois at Urbana-Champaign
The theory for realistic mathematics education establishes that realistic contexts framing mathematics problems provide opportunities for guided reinvention. I use data from a geometry study group to examine teachers' understandings of what constitutes a realistic context for using students' prior knowledge during a Lesson Study cycle. Overall, participants increased their attention to students' prior knowledge and identified relevant contexts for their students when planning and implementing a lesson about finding locations on a map.

What Might RME Look Like in a First Year Algebra Course?
Pamela Weber Harris, The University of Texas at Austin
In our project, Focus on Algebra, we designed 3 sets of sequenced tasks to teach major first year algebra topics based on algebra landscapes of learning and the lesson structures of truly problematic situations and problem strings. This session will briefly describe our project, engage participants in tasks, show video of students and teachers at work, discuss design principles, and offer suggestions for further research and collaboration.

FaSMed: an approach to assessment that helps teachers
Marja van den Heuvel-Panhuizen & Mieke Abels, Freudenthal Institute, University of Utrecht
FaSMEd is short for Formative Assessment in Science and Mathematics Education. In the Dutch part of the EU-FaSMEd project a Digital Assessment Environment (DAE) is being developed for mathematics education in grades 5 and 6 of primary school. The DAE not only makes visible whether students can solve correctly key problems linked to the standards, but also which optional auxiliary tool they used to reach their solutions. In this way, the DAE provides the teachers with important clues as to how students can be helped best to (further) develop an understanding or skill. The mathematical domains selected for the FaSMEd project from the national curriculum are those with which students have considerable difficulties, being: fractions, percentage, and the metric system. In the workshop, we would like you to explore the features of the DAE in an interactive way and to share the first results of our research.

Designing and sharing materials to promote student interaction and active learning
Yuko Ichikawa and Mari Morimoto
National Institute of Technology, Tokyo College & Akita College
In order to implement active learning in math classes, we have been preparing materials outside of standard lectures notes. While this requires considerably more time and effort, we have successfully attained this goal with handouts that are specially designed such that students are able to conduct self-study alongside textbooks. This method has also been utilized together with GeoGebra programs.

Modeling where it matters most — Disrupting pervasive patterns in math education
Kara Imm, Math in the City, City College of New York
I will describe findings from a recent teaching experiment, designed to study how mathematical modeling impacted conceptual learning as well as classroom agency for urban girls of color. The study was situated to investigate (and disrupt) the pattern of "endless algebra" in high school mathematics. This will be of particular interest to those thinking about: incorporating students' lived experiences into classroom activities, RME in secondary mathematics, and the specific aspects of modeling that supported students.

Designing Number Strings — The Essential Aspects of Context and Models
Kara Imm, Math in the City, City College of New York
Number strings, originally made accessible to American audiences by Fosnot and Dolk in their series *Young Mathematicians at Work*, were originally conceived at the Freudenthal Institute. Their popularity has resulted in a series of other noteworthy numeracy routines (e.g., number talks, math talks, problem strings). In this session I will begin by showing how these routines are related, to what extent they have moved from "bare number" work towards mathematical relationships in situated contexts, and what may explain their current popularity. Next, and more importantly, I will demonstrate how the use of both context and didactic models provides a critical "anchor" to all students that is often missing from the design of other routines.

Leveraging a Dynamic Computer Environment to Foster Secondary Students' Shifts from Variational to Covariational Reasoning: The Ferris Wheel
Heather Lynn Johnson, Peter Hornbein & Sumbal Azeem, University of Colorado Denver
Although covariational reasoning is essential for secondary students, little is known about its development. Reporting results from a design experiment study with five ninth grade students, in which we implemented a Ferris wheel Dynamic Computer Environment and related tasks, we document a student's shift from variational to covariational reasoning. We recommend that instructional tasks provide students opportunities to investigate individual quantities in the process of changing, then to form and interpret relationships between changing quantities

Designing teaching and learning resources for post-compulsory mathematics
Elizabeth Kimber and Anna Baker
Cambridge Mathematics Education Project, University of Cambridge
The Cambridge Mathematics Education Project (CMEP) is developing rich tasks for teaching A-level (post-compulsory) mathematics. The project is funded by the UK Department for Education and run by members of the University of Cambridge Faculty of Mathematics. The CMEP design process began by organizing A-level mathematics content thematically. Teaching

resources are being developed to highlight connections and support mathematical thinking. Many CMEP tasks require students to move between algebraic and graphical representations, to generate their own examples, and evaluate statements. The project recognizes that many teachers face challenges in implementing ambitious student-centred approaches. To support the effective use of these rich tasks and to contribute to a range of professional development programs, CMEP is collaborating with the University of Cambridge Faculty of Education to develop support materials. These materials, which include video clips and teacher notes, are being iteratively evaluated and developed in A-level classrooms. In this session we will describe the project site, share some resources to illustrate our approach and discuss ideas emerging from our work with teachers at our partner schools.

From Design to Classroom Implementation: Interactive Simulations for Teaching and Learning Mathematics
Amanda McGarry, University of Colorado Boulder
PhET Interactive Simulations use context, representation, interactivity, implicit scaffolding, and feedback to provide students with an engaging setting for learning mathematics. We will examine these design elements in an upcoming simulation, Function Builder, which include the function machine model and multiple representations of a function. Participants will explore the Balancing Act, Fractions Intro, and Area Builder simulations, reflect on classroom video, and discuss how simulations can support the development of mathematical thinking.

Effectiveness of Personalizing Instructive Software for Rational Numbers: A Randomized Controlled Trial
Brent Milne & Terry Wyberg, WootMath Inc & University of Minnesota
We will discuss the development of an online instructional sequence for rational numbers and technology for adaptively sequencing materials for each student. Both designs have been informed by principles of RME. We will discuss how our research relates to RME and also draw connections between RME and research from the Rational Numbers Project. The RCT provides evidence for the effectiveness of the adaptive instructional design and, indirectly, for the underlying RME principle of guidance.

Emergent modeling: From chains of signification to cascades of artifacts
Frederick Peck, Department of mathematical sciences, University of Montana
Emergent modeling is a key design principle in RME. Emergent modeling is often described using a construct from semiotics called a chain of signification. I show that chains of signification are inadequate to describe both the process and product of emergent modeling. To overcome these inadequacies, I introduce a new construct called the *cascade of artifacts*. I conclude with implications for research and design.

Peer-Assisted Reflection: Developing Mathematical Practices Through Iterative Cycles Of Reflection And Revision
Daniel Reinholz & Dimitri Dounas-Frazer, University of Colorado Boulder
This session focuses on Peer-Assisted-Reflection (PAR), a cycle of activities that requires students to: (1) work on meaningful problems, (2) reflect on their own work, (3) analyze a peer's work and exchange feedback, and finally (4) revise their work based on insights gained throughout this cycle. Participants will learn the theory behind PAR, how to support students to

give productive feedback, and gain access to practical materials they can use in their own classrooms.

Inverse Functions from an Emergent-Modeling View, Presented in the Digital Mathematics Environment (DME)
Kevin J. Reins, University of South Dakota
Utilizing the emergent-modeling design heuristic of RME and the modeling cycle of the CCSSM, an experience was shaped to assist students' abstraction of the mathematics in a personal, meaningful way. The session will explore the activity, a byproduct of a Lesson Study with prospective secondary mathematics teachers, in the Digital Mathematics Environment (DME) and how this emergent-modeling viewpoint can support and prompt new levels of student mathematical understanding.

Framing a Geometry Trajectory on RME Principles for Methods and Content Courses for Undergraduate Preservice Teachers
Jacqueline Sack & Judith Quander, University of Houston Downtown
The Common Core presents a relatively coherent Geometry flow across all grade levels. For high school they suggest extensive instructional approaches, but very little for K-8. This proposed trajectory, based on Van Hiele levels, begins with hands-on geometric activities appropriate for grades 4-12. These develop coherently through guided discovery. Examples of student work will be shared including those showing the detrimental effects of using limiting constructs or of treating concepts in isolation rather than coherently.

A Study of Mathematics Education and Culture: An Analysis of Students' Works of Graphing Calculator Art
Takehiro Tsubokawa, National Institute of Technology, Fukui College
The creation of works of graphing calculator art potentially enhances their appreciation of the relevance of mathematics to the real world. In light of our findings, students' choices of subject matter are influenced by their cultural and social backgrounds. In this presentation, I examine both mathematical elements and cultural elements by analyzing approximately 80 prize-winning works and the winners' comments during the years 2007-2014.

Developing 3rd Grade Children's 3D Visualization and Numeration Skills using Intuitively Accessible Models
Irma Vazquez & Jacqueline Sack, Houston ISD & University of Houston Downtown
This guided re-invention framework uses the seven Soma puzzle figures that children can handle; conventional 2D pictures of Soma assemblies; and, abstract representations including numeric top-view diagrams. We also use a digital interface, Geocadabra Construction Box, that dynamically simulates these representations. We will engage participants in selected activities, in addition to seeing pictures and video clips of students' work. These include strong numeration connections that go far beyond those expected in the current Common Core.