Math and sciences: different subjects, a common goal for STEM

Freudenthal Institute
for Science and Mathematics Education

Henk van der Kooij

Freudenthal Institute, Utrecht University

h.vanderkooij@uu.nl

Goals for math education (re-phrased likewise for sciences)

1. To prepare for (critical) citizenship
2. To prepare for future work
3. To prepare for further learning

But what about connecting individual school subjects?

STEM goals

1. Integration vs isolation of school subjects
2. More students going for STEM-studies
3. Applications based learning

Focus today: integration of or better: coherence between school subjects in 9-12 education

Intermezzo: Policies and Politics

We believe, after examining the findings of cognitive science, that the most effective way of learning skills is "in context," placing learning objectives within a real environment rather than insisting that students first learn in the abstract what they will be expected to apply.
(*SCANS, **1991**, USA*)

Mathematical competence is the ability to develop and apply mathematical thinking in order to solve a range of problems in everyday situations. Building on a sound mastery of numeracy, the emphasis is on process and activity, as well as knowledge. Mathematical competence involves, to different degrees, the ability and willingness to use mathematical modes of thought (logical and spatial thinking) and presentation (formulas, models, constructs, graphs, charts).
(*Key competence 3, **2007**, EU*)

Experiences so far

1. TWIN project: math and science in vocational engineering courses (age 16-20), NL
2. TechMAP: COMAP (Sol Garfunkel), USA modules math and science for HS
3. SaLVO project (grades 8-12), NL Coherence mainly in math and physics
4. Primas project (K-12,EU) focus on Inquiry Based Learning (STEM-focused)

Main focus today:

How to better connect math and science in teaching/ learning?

Math and Sciences:

Important foci

1. Variables vs quantities

2. proportionality: a key concept in sciences!

3. Graphical representations

4. numbers: exact vs imprecise

Just snapshots (appetizers?) to get us aware of chances and challenges

1. Variables vs quantities

Variables and parameters in math are placeholders for ‘just a range of allowed numbers’.
They are meaningless; names can be freely exchanged.

Some conventions:
variables are preferably x (independent) and y (dependent)
Parameters a, b, m, n, …

Quantities and parameters in science are meaningful:

The names indicate what they stand for: a physical entity

Parameters often have a role of ‘quality indicator’ for the relationship between two quantities

1. Variables vs quantities

Example 1:

$$T = 2\pi \cdot \sqrt{\frac{l}{g}} \qquad \text{versus} \qquad f(x) = \mathrm{c} \cdot \sqrt{x}$$

Many students in vocational training (TWIN project) were able to work with the equation-in-context, while they felt stuck at the formal, abstract one

“I don’t know what x and $f(x)$ stand for; it has no meaning to me. So what do you expect me to do?”

Transfer comes by no way naturally but needs a lot of attention (contextual vs abstract)

Intermezzo: Situated abstraction

Practitioners at work (including scientists?) do use *situated abstraction* in which local mathematical models and ideas are used that are only partly valid in a different context because they are connected to anchors within the context of the problem situation itself (Hoyles & Noss, UK)

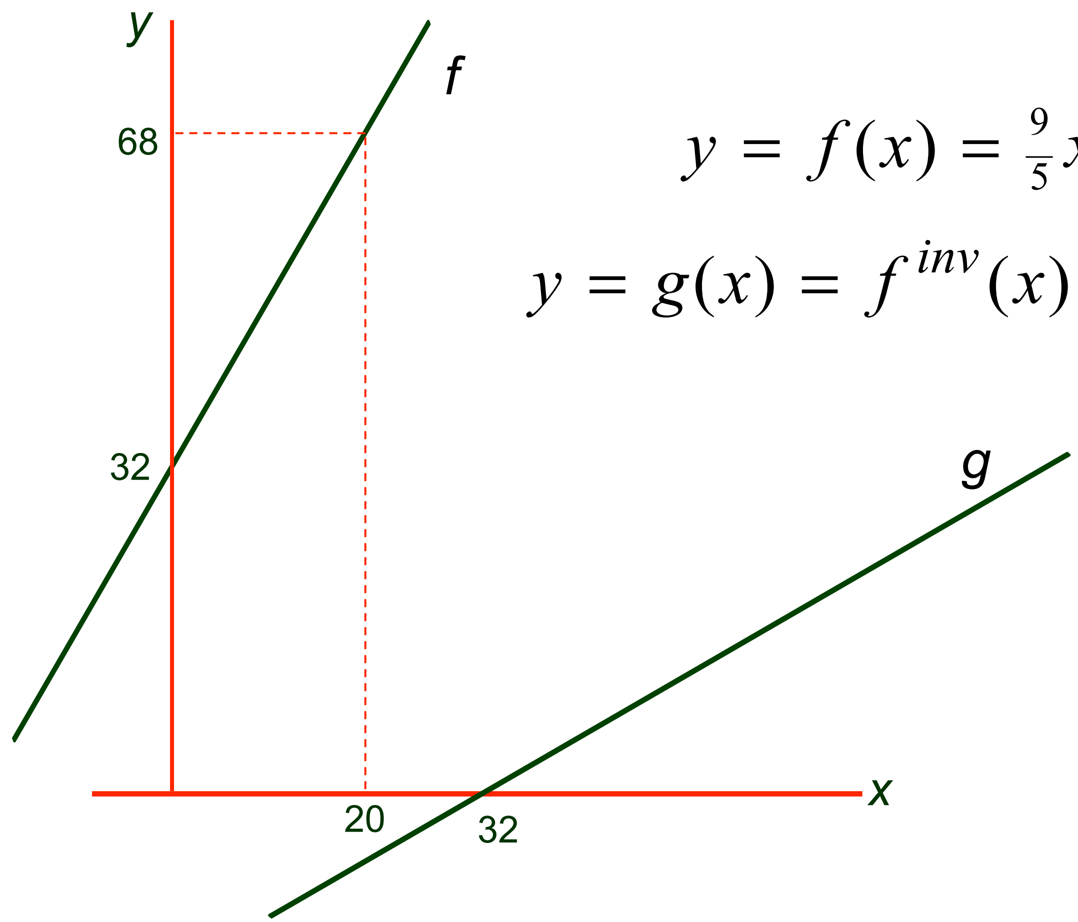
$$F = m \cdot a \qquad \text{and} \qquad U = I \cdot R$$

- The scientific context is of influence on the way in which mathematical activities take place
- Transfer of mathematical knowledge (even within scientific contexts) is not self-evident

1. Variables vs quantities

Example 2: Inverse functions (math)

Given a straight line through (0, 32) and (20,68). Find $y = f(x)$ and also $y = g(x) = f^{\text{inv}}(x)$



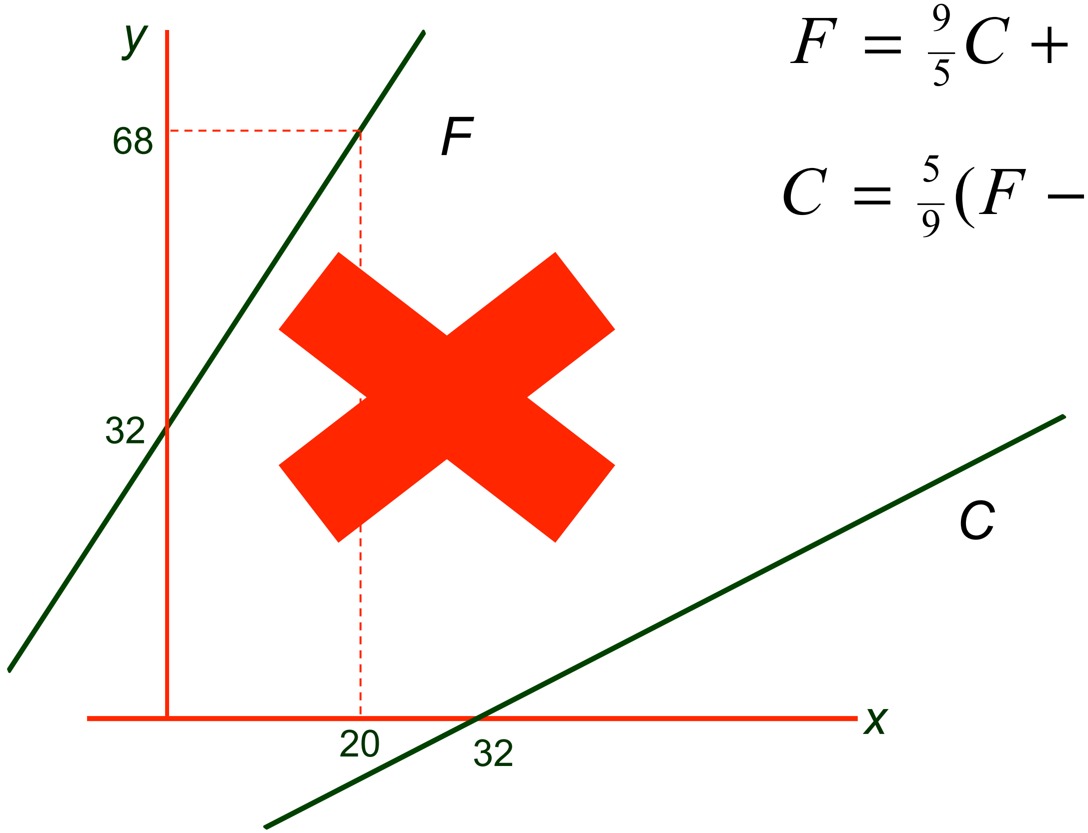
$$y = f(x) = \tfrac{9}{5}x + 32$$

$$y = g(x) = f^{inv}(x) = \tfrac{5}{9}(x - 32)$$

1. Variables vs quantities

Example 2: Inverse relationships (science)

Conversion from C(elsius) to F(ahrenheit) is determined by:
68°*F* = 20°C and 32°*F* = 0°C



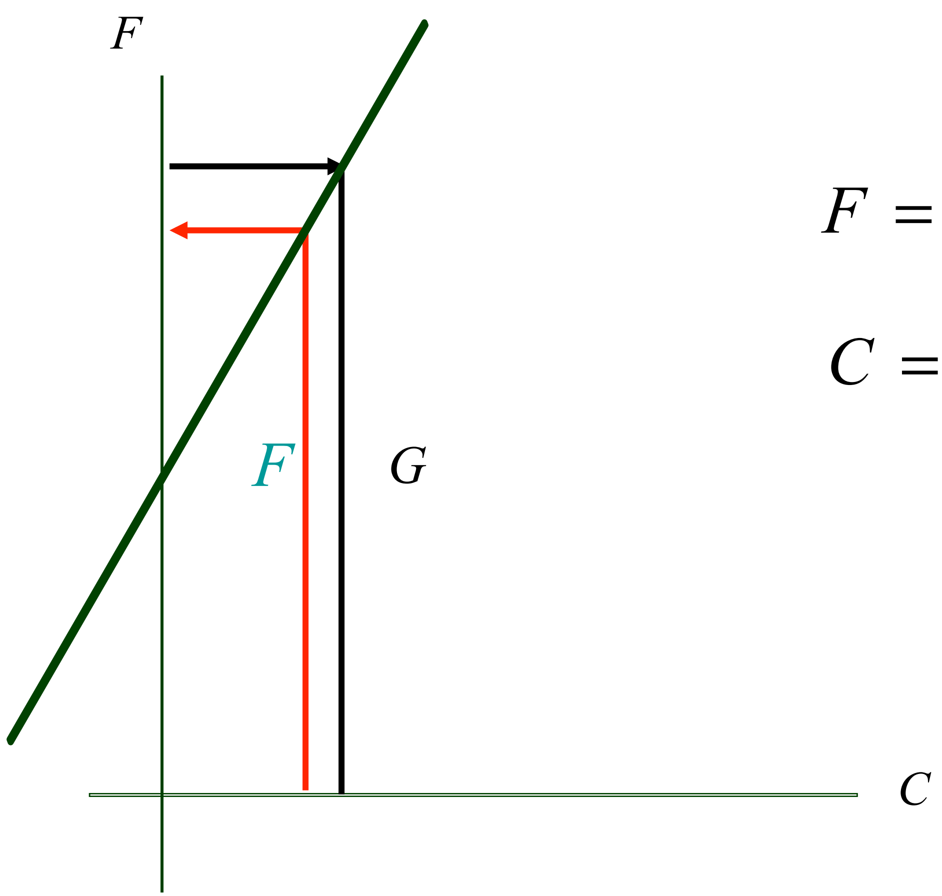
$$F = \tfrac{9}{5}C + 32$$

$$C = \tfrac{5}{9}(F - 32)$$

1. Variables vs quantities

Example 2: Inverse relationships (science)

Just one graph: read the graph from horizontal to vertical (relationship) and from vertical to horizontal (inverse relationship)



$$F = \tfrac{9}{5}C + 32$$

$$C = \tfrac{5}{9}(F - 32)$$

2. proportionality: a key concept in sciences!

Polynomials are hardly found in science ….
(sorry mathematicians… factoring is not very important in -basic- science)

But proportional relationships are everywhere

proportional relationships are, mathematically spoken, power functions like $y = c \cdot x^a$

But often interpreted and used in a different way…

Most important values for a are $a \in \{1, -1, 2, -2, \frac{1}{2}, -\frac{1}{2}\}$

2. proportionality: a key concept in sciences!

A short selection of proportional relationships in science

$$s = v \cdot t$$

$$p = \frac{n \cdot R \cdot T}{V} \quad \text{or} \quad p \cdot V = n \cdot R \cdot T$$

$$F = G \cdot \frac{m_1 \cdot m_2}{r^2}$$

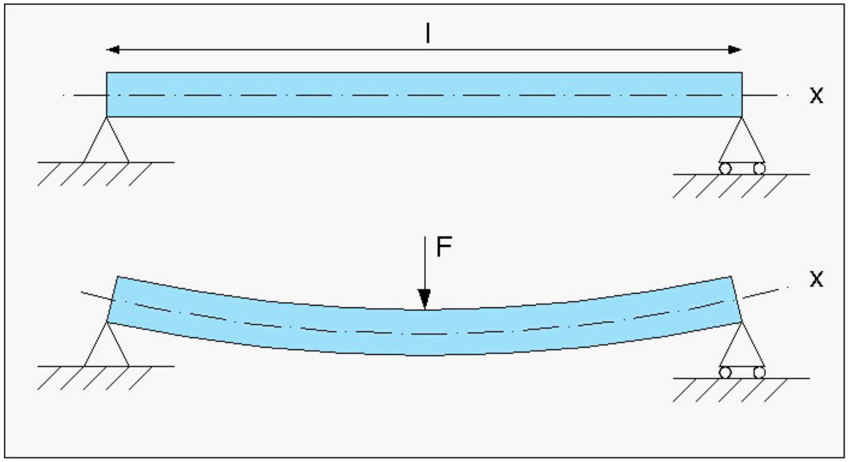
$$f = \frac{c \cdot F \cdot l^3}{E \cdot w \cdot h^3} \cdot 9.81$$

Intermezzo:

Bending of a beam

A beam with length l, width w and height h (all in mm) is supported at both ends.

Due to a force F (in N), the beam will bend.



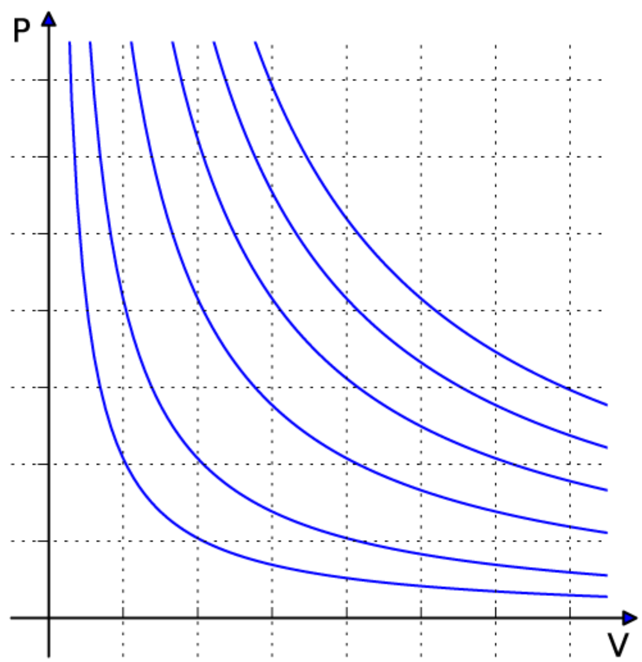
The flexion f (in mm) in the middle of the beam can be found using this complex looking formula:

$$f = \frac{c \cdot F \cdot l^3}{E \cdot w \cdot h^3} \cdot 9.81$$

Activity: 'bending of a beam'

3. Graphical representations

- In mathematics the study of graphs is mostly restricted to $y = f(x)$ (2D graphs) or $z = f(x, y)$ (3D graphs)
- In sciences relationships often involve more than 2 or 3 variables.



Isothermal process (T = constant)

3. Graphical representations

An interesting difference between math and physics:

Graphing proportionality $y = c \cdot x^a$

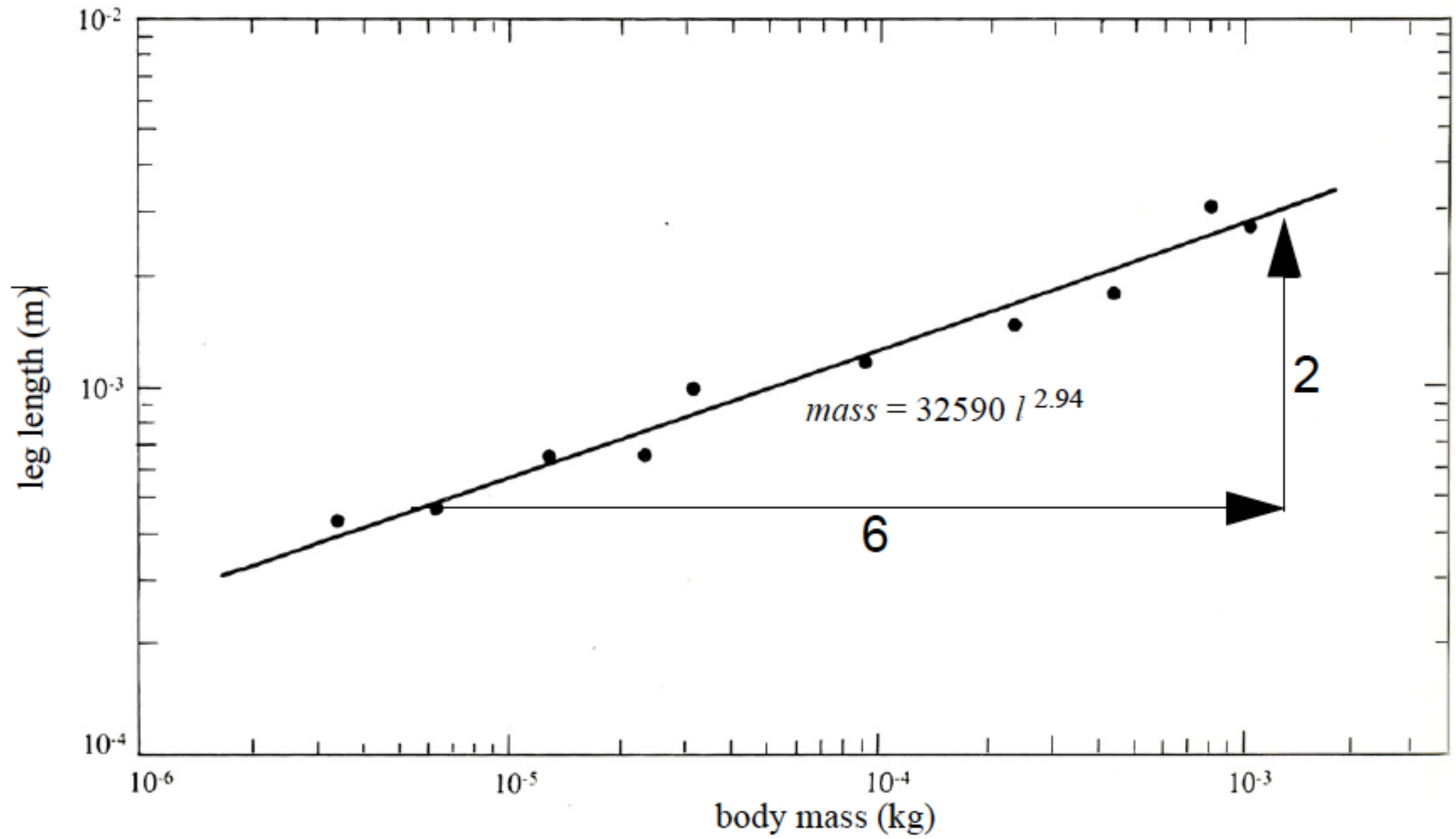
The often used method in mathematics:

$$y = c \cdot x^a \rightarrow \log(y) = \log(c) + a \cdot \log(x) \rightarrow p = d + a \cdot q$$

Using log-log paper, every proportional relationship will appear as a straight line, where the slope of the line is the exponent a

3. Graphical representations

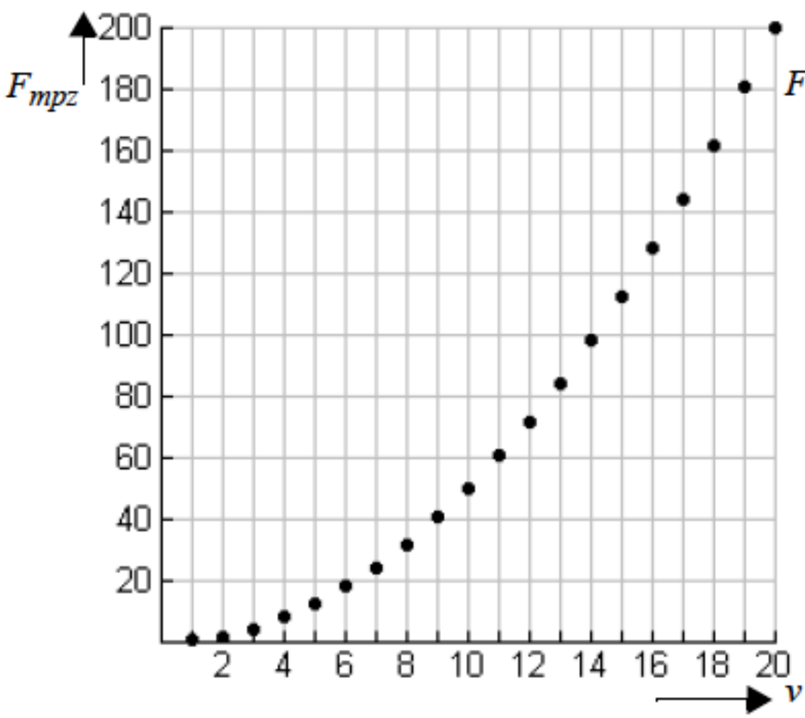
An example: relationship between leg length and body mass of a number of cockroaches

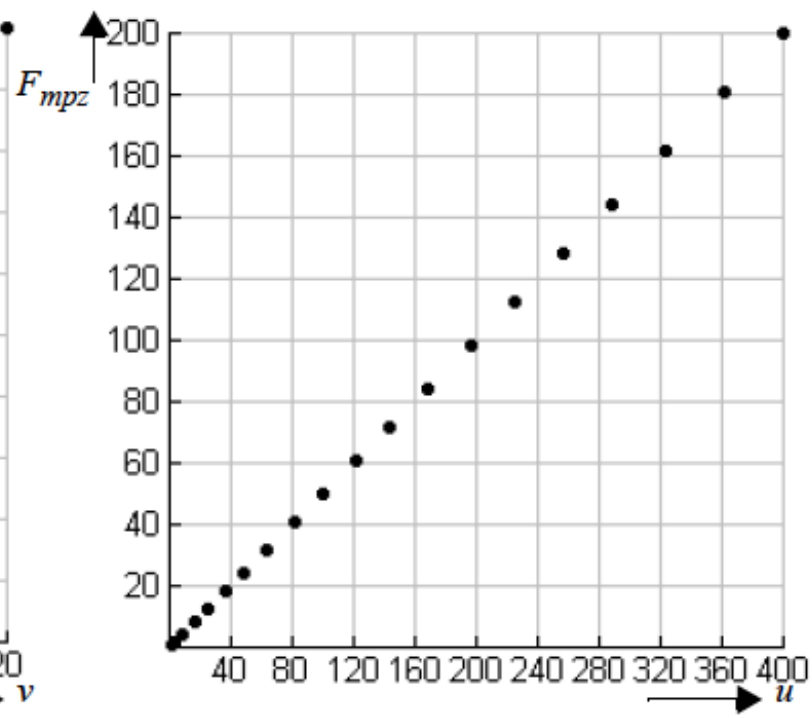


The given formula is the result of 'best fitting line'. But mass is proportional to volume, so the exponent 3 instead of 2.94 seems better

3. Graphical representations

Physicists (at least in the Netherlands in HS and college) prefer what they call ‘coordinate transformation’. If you think that one quantity is proportional to the square of another quantity (like in $F_{mpz} = c \cdot v^2$) you put the squares of v (or $u = v^2$) on the horizontal axis.
The graph then becomes a straight line.





3. Graphical representations

My thoughts about this (hopefully triggering for some discussion)

- The mathematical method (using log-log graphs) is very general and works for all cases of proportionality

- The name ‘coordinate transformation’ is for mathematicians misleading because it refers to Linear Algebra in a completely different setting. I would prefer ‘substitution method’ because substituting $u = v^2$ makes the relationship linear $F_{mpz} = c \cdot v^2 = c \cdot u$

- Why do physicists prefer ‘coordinate transformation’ above using ‘log-log’ . I am curious to know…

4. Numbers: exact and imprecise

In math class, numbers are (always) exact

In science class, numbers are (always) results of measurement or a production process and therefore not exact…

In math 2 and 2.0 and 2.000 are all just 2
and $2.0 \leq x \leq 5.0$ is seen as $2 \leq x \leq 5$

In 'science thinking'
- 2 is any value between 1.5 and 2.5
- 2.0 is any value between 1.95 and 2.05
- 2.000 is any value between 1.9995 and 2.0005

and $2.0 \leq x \leq 5.0$ is impossible, because the interval is not 'closed' it is something like $1.95 < x \leq 5.05$ given that the data have 2 significant digits

4. Numbers: exact and imprecise

An example of working with imprecise numbers

In natural sciences, the coefficient of linear thermal expansion α is a property of materials that indicates the factor with which the material increases in length (relative to the initial length l) for each degree of temperature rise: $\alpha = \frac{\Delta l}{l}$

The coefficient for cubic thermal expansion is then $\gamma = 3\alpha$

Explanation:

$$\gamma = \frac{(l+\Delta l)^3 - l^3}{l^3} = \frac{3l^2\Delta l + 3l\Delta l^2 + (\Delta l)^3}{l^3} = 3 \cdot \frac{\Delta l}{l} + 3\left(\frac{\Delta l}{l}\right)^2 + \left(\frac{\Delta l}{l}\right)^3$$

Neglecting higher powers of $\frac{\Delta l}{l}$ (small compared to $\frac{\Delta l}{l}$) results in

$$\gamma = 3\alpha$$

5. Quantities: dimensions and units

A number in science is never 'just a number'

A number represents a numerical value of a quantity, so it has both 'dimension' and 'unit of measurement'

Some dimensions in the SI-system are:
L for *length*. T for *time*, M for *mass* and others.
Standard units of measurement: L in m, T in sec, M in kg

[distance travelled]=[distance]=[width]=[height]= L

[velocity] = LT^{-1}=L/T

[Force] = [mass x acceleration]= MLT^{-2}

5. Quantities: dimensions and units

An example from physics

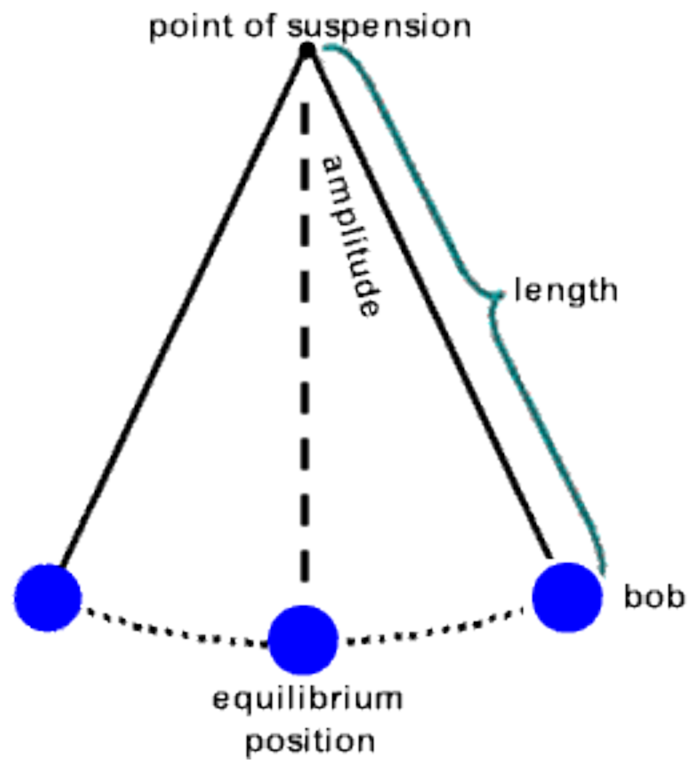
The period T of a pendulum with length l is described by the formula:

$$T = 2\pi \cdot \sqrt{\frac{l}{g}}$$

[period] = T^{-1}
[l] = L
[g] = LT^{-2}

[l / g] = L / (LT^{-2})= T^2

$$[\text{period}] = \left[\sqrt{\frac{l}{g}}\right] = \left[\sqrt{T^2}\right] = [T]$$



5. Quantities: dimensions and units

An example from math, showing that considering dimensions can also make sense in math education

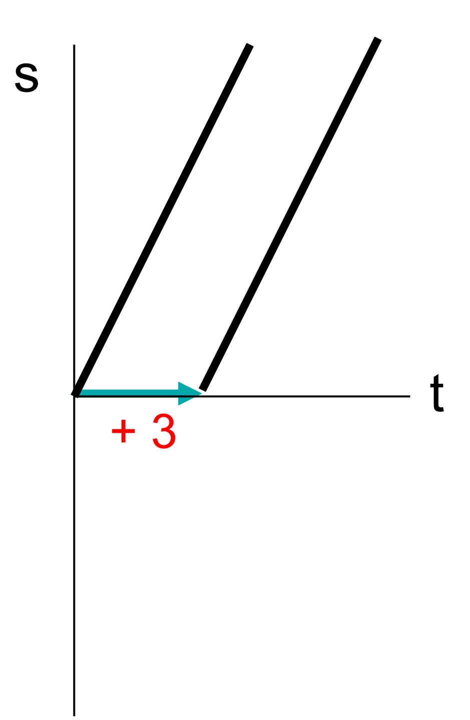
Algebraically, the following two equations are identical:

$s = 2(t - 3)$ and $s = 2t - 6$

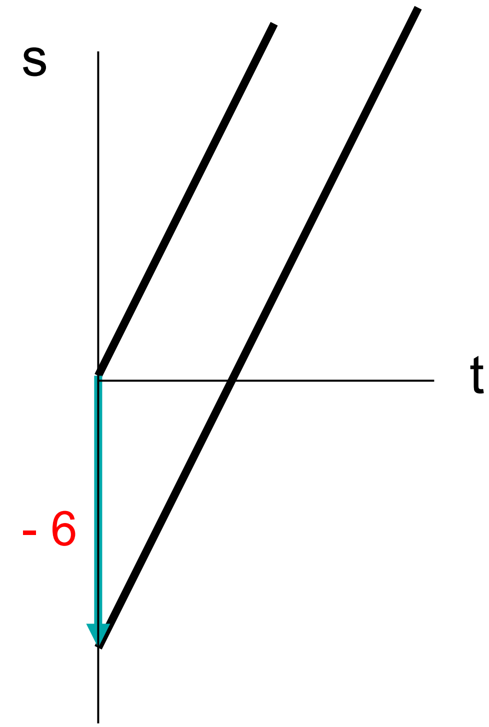
If the context is:
One person (A) is walking at constant speed 2m/s
Another person (B) starts 3 seconds later at the same speed from the same starting position …

The two equations are not the same anymore!!!

$s = 2 * (t - 3)$ and $s = 2t - 6$



distance = speed * time



distance = distance - distance

So, 3 represents *time* and 6 represents *distance*

To conclude

These ‘snapshots’ were meant to open the minds of math- and science people for better awareness of what we have in common and how we differ

For the sake of STEM, but also for our students, a good dialogue between math- and science educators is needed

This was the main message I wanted to communicate…

I hope at least some issues triggered you…
More detailed discussions can be found in:
‘Algebra in Science and Engineering’,
chapter 9 in ‘Algebra in secondary education’